Consumer Search and Price Competition*

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Abstract

We consider an oligopoly model in which consumers engage in sequential search based on partial product information and advertised prices. By applying Weitzman's (1979) optimal sequential search solution, we derive a simple static condition that fully summarizes consumers' shopping outcomes and translates the pricing game among the sellers into a familiar discrete-choice problem. Exploiting the discrete-choice reformulation, we provide sufficient conditions that guarantee the existence and uniqueness of market equilibrium and analyze the effects of preference diversity and search frictions on market prices. Among other things, we show that a reduction in search costs raises market prices.

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1 Introduction

We present an oligopoly model in which consumers sequentially search for the best product based on partial product information and advertised prices. A key distinguishing feature from

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traditional consumer search models is the observability of prices before search. Consumers still face a non-trivial search problem, because they do not possess full information about their values for the products. In this environment, prices affect each seller's demand not only through their effects on consumers' final purchase decisions, but also through their effects on consumer search behavior. We study how the presence of the latter channel affects sellers' pricing incentives and what its economic consequences are. In particular, we investigate the effects of preference diversity and search frictions on market prices.

Our investigation is mainly motivated by some dramatic changes in retail markets due to the rapid growth of the Internet. The Internet has significantly lowered the cost of collecting price information. Now it is common to check prices online and visit stores only to get hands-on information and/or finalize a purchase.[1] This change in consumer behavior has various economic implications. Classic sales tactics that exploit consumers' lack of price information are likely to become obsolete, which will force firms to develop new pricing and advertising strategies. Accordingly, it will become more important in consumer protection to ensure that firms provide enough and accurate information not only at stores, but also on the Internet. Our model can serve as a basic framework to address these and related issues.

In addition, electronic commerce (i.e., online shopping) has already come into our daily lives and is expected to play an increasing role in the economy.[2] Naturally, the literature on electronic commerce is growing fast in various directions. This paper makes a potentially significant contribution to the literature, because our model captures some salient features of online marketplaces and price comparison websites and, therefore, can yield meaningful insights about how they work. Online shopping typically unfolds as follows: a consumer begins by searching with a specific keyword or choosing an appropriate category. She then faces a summary webpage that displays multiple items with price and brief product descriptions. She clicks a certain set of items, collects more detailed information, and makes a final purchase decision. Our model describes this consumer behavior particularly well.

Similar models have been studied in three recent papers, Armstrong and Zhou (2011), Shen (2015), and Haan et al. (2017). All three papers analyze a symmetric duopoly envi-

[1]According to "The 2011 Social Shopping Study" by *PowerReviews*, 57% of respondents compared prices through Google Shopping. In addition, while shopping in physical stores, 36% of consumers checked Amazon prices and 36% looked for other retailers' prices.

[2]According to *Internet Retailer*, in the U.S., total e-commerce sales were $349.25 billion in 2015 and grew by 14.4% in 2016. Moreover, they accounted for 41.6% of all retail sales growth in 2016.

ronment but consider different correlation structures for consumers' prior and match values. Both prior and match values are negatively correlated between the products in Armstrong and Zhou (2011), whereas both are independent in Haan et al. (2017). Shen (2015) examines an intermediate case where each consumer's prior values are negatively correlated, while her match values are independent, between the two products. Our model adopts the same independence structure as Haan et al. (2017) but allows for general market structure and asymmetric sellers.

It is well-recognized that such consumer search models do not admit tractable characterization. The main difficulty lies in the fact that consumer search behavior is complicated and hard to summarize. Even with two sellers, there are three different paths through which consumers purchase from each seller: some consumers purchase immediately from their first visit. The others visit both sellers and purchase either from their first visit or from their second visit. The number of purchase paths increases factorially fast as the number of sellers increases. This is likely to be the reason why all previous studies have restricted attention to the symmetric duopoly case.

The main breakthrough of this paper is to derive a tractable condition that summarizes consumer search outcomes, which we refer to as the eventual purchase theorem.[3] Given prices, each consumer faces a non-stationary sequential search problem, to which an elegant solution by Weitzman (1979) applies. Weitzman's solution consists of the optimal search order (in which order to visit the sellers) and the optimal stopping rule (when to purchase or take an outside option) and, therefore, fully describes optimal search *behavior*. For our purpose, however, it suffices to know optimal search *outcome*, that is, which product each consumer eventually purchases. We show that each consumer's eventual purchase decision can be fully inferred from a static discrete-choice condition with appropriately defined values based on her prior and match values and search costs. The condition reveals that the distortions imposed by search frictions on consumers' purchase decisions (i.e., the extent to which consumers' purchase decisions under sequential search differ from those under perfect information) take a simple and systematic form.

The eventual purchase theorem implies that the pricing game among the sellers can be reformulated as a discrete-choice problem. The reformulation is useful by itself, because it allows us to analyze the model without keeping track of different purchase paths. Fur-

[3] We discuss two relevant studies, Armstrong and Vickers (2015) and Armstrong (2016), in Section 2.

thermore, it enables us to apply rich existing results on discrete-choice models to our search model. To begin with, it is infeasible to establish the existence of market equilibrium (let alone its uniqueness) with an elementary method: the sellers' best response functions are hard to derive and, even if they can be derived, do not behave well in general (see Haan et al., 2017). However, based on a general result in the discrete-choice literature, we provide sufficient conditions under which there exists a unique market equilibrium, which is in pure strategies, in our model.

We provide a comparative statics result, which is not only useful in our search model, as explained shortly, but also contributes to the discrete-choice literature more generally. The result is concerned with a systematic relationship between preference diversity (product differentiation) and equilibrium prices. Within the random utility framework of Perloff and Salop (1985), it has remained an open question what is an appropriate measure of preference diversity, that is, what distributional changes cause market prices to rise. Our result provides an answer to this question: in the symmetric environment, market prices increase if the value distribution becomes more dispersed in the sense of *dispersive order* and does not decrease too much in the sense of first-order stochastic dominance. Importantly, if there is no consumer outside option, as assumed in Perloff and Salop (1985) and many subsequent studies, then the second requirement becomes vacuous and, therefore, dispersive order alone dictates market prices.[4]

As a methodological contribution, we develop an indirect approach to comparative statics regarding search frictions, which is based on our discrete-choice reformulation and the comparative statics result for general discrete-choice models mentioned above. In our model, the effects of various changes in search frictions can be summarized by their effects on dispersion of the induced discrete-choice distributions. Combining this with our general comparative statics result, we can indirectly learn about the effects of search frictions on market prices, without analyzing their effects on different purchase paths. This allows us to obtain a set of results that are otherwise difficult to obtain in our general environment.

We show that in the symmetric environment, market prices fall with search costs, which is opposite to the classical result on consumer search.[5] As the value of search decreases,

[4]We introduce a particularly relevant contribution by Zhou (2017) and some other economic applications of dispersive order in Section 2.

[5]Haan et al. (2017) obtain this result in the duopoly environment with no consumer outside option. As elaborated in Section 6.2, our contribution is to show that this result holds in a more general environment

each consumer is less likely to leave for another seller and, therefore, more likely to purchase from the current seller. Each seller then has an incentive to extract more surplus from visiting consumers and, therefore, charges a higher price. This is the main mechanism behind the opposite result in the literature. However, it crucially depends on the assumption of unobservable prices, which implies that the sellers cannot influence consumer search behavior. In our model, the sellers compete in prices to attract consumers. When the value of search falls, it becomes more important for the sellers to attract consumers in the first place, which intensifies price competition and leads to lower prices.

In contrast, improving pre-search information quality has an ambiguous effect on market prices. We show that in the symmetric Gaussian environment, providing more precise product information before consumer search increases market prices if and only if the number of sellers is above a certain threshold. There are two opposing effects. On the one hand, it reduces consumers' incentives to explore more products, which, as above, intensifies price competition among the sellers. On the other hand, consumers' preferences before search (prior values) become more dispersed, which relaxes price competition. We demonstrate that the latter effect dominates the former and, therefore, providing more product information before search increases market prices if there are sufficiently many sellers.

The rest of the paper is organized as follows. We explain how our work is related to various strands of literature in Section 2 and introduce the formal model in Section 3. We analyze consumers' optimal shopping problems in Section 4 and characterize market equilibrium in Section 5. We study the effects of preference diversity and search frictions on market prices in the symmetric environment in Section 6 and conclude in Section 7. All omitted proofs are in the appendix.[6]

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by adopting the indirect approach explained above. As argued and explained in more detail by Haan et al. (2017), this result is consistent with some recent empirical findings. For example, Moraga-González et al. (2015) structurally estimate a consumer search model using data from the Dutch automobile market, where price information is believed to be easily accessible, and find that reducing inspection costs led to higher prices for some car models. See also Koulayev (2014), Pires (2015), and Dubois and Perrone (2015) for similar and related findings in different markets.

[6] In an earlier version, we provided two sets of results for the case of asymmetric sellers. The main economic messages were (i) that Weitzman values do not provide enough guidance about price rankings in general (i.e., a seller with a lower Weitzman value may announce a higher price), and (ii) that price dispersion (difference) may reduce and a lower cost seller's profit can increase as search costs rise.

2 Related Literature

This paper joins a growing literature on ordered search, which investigates the effects of (both exogenous and endogenous) search order on market outcomes and various ways sellers or intermediaries influence consumer search behavior (order). See Armstrong (2016) for a comprehensive and organized introduction of the literature and several useful discussions. In light of this literature, we consider the case where each consumer's search order is fully endogenized and each seller influences search behavior through the choice of his price, which is arguably the most basic instrument.

Our eventual purchase theorem was anticipated by Armstrong and Vickers (2015) and independently discovered by Armstrong (2016). Armstrong and Vickers (2015) provide a necessary and sufficient condition for a multiproduct demand system to have a discrete-choice micro-foundation and show that the condition holds in a consumer search model in which prices are observable and visiting seller 1 is costless, while visiting seller 2 is costly. Our theorem can be interpreted as an application of their general result, and a closed-form solution, to a more general environment. Armstrong (2016) derives an effectively identical condition to ours but uses the result to motivate and discuss various ideas on ordered search, rather than exploiting it to study a specific consumer search model as we do in this paper.

Dispersive order, which we utilize in order to establish comparative statics regarding search frictions, has been found useful in several other economic applications. Zhou (2017) independently discovers an almost identical result to our dispersive order result and uses it to study the effects of bundling in the Perloff-Salop framework. Our result is slightly more general than his, because we allow for consumers' outside options: our result reduces to his result when consumers have no outside option. Ganuza and Penalva (2010) show that dispersive order is useful in evaluating the effects of information disclosure in symmetric private-value second-price auctions: as the distribution of conditional expectations becomes more dispersed, efficiency always improves, but seller surplus does not necessarily increase. Bhaskar and Hopkins (2016) consider a frictionless two-sided matching market with noisy premarital investments and show that dispersive order can be used to predict which side over- or under-invest relative to the efficient level.

As explained in Section 1, one of the most important economic implications of observable prices is that market prices decrease with search costs. Several recent papers identify other mechanisms that yield the same result. For example, Zhou (2014) considers a multi-product search model and shows that the joint search effect (that decreasing one product's price raises the other product's demand as well) can outweigh the usual market power effect and, therefore, prices can decrease with search costs. Moraga-González et al. (2017) introduce heterogeneous search costs and consumer outside option into the model of Wolinsky (1986) and demonstrate that the extensive search margin (how many consumers choose to search in the first place) may induce prices to decrease as search costs increase in the sense of first- or second-order stochastic dominance. See also Garcia et al. (2015), who study a model in which both consumers and retailers search within a vertical industry structure, and Shelegia and Garcia (2015), who analyze an observational learning model in which each consumer observes the purchase decision of the immediate predecessor before search.

There is a fairly large literature on directed search, which has been developed mainly in the labor context: see Moen (1997) and Burdett et al. (2001) for some seminal contributions. The main economic problem in those models is congestion among searchers (i.e., coordination frictions) resulting from capacity constraints (each firm has one vacancy, or each seller has unit supply), which is absent in our model. In addition, in most directed search models, each searcher faces a stationary or static search problem, which does not necessitate a new and elaborate analysis of optimal sequential search behavior as we provide here.

Our paper is also related to two strands of literature on electronic commerce. First, several papers have developed an equilibrium online shopping model. For example, Baye and Morgan (2001) present a model in which both sellers and consumers decide whether to participate in an online marketplace, while Chen and He (2011) and Athey and Ellison (2011) present a model that combines position auctions with consumer search. Our paper is unique in that the focus is on consumer search within an online marketplace. Second, a growing number of papers draw on search theory to study online markets. For example, Kim et al. (2010) develop a non-stationary search model to study the online market for camcoders. De los Santos et al. (2012) test some classical search theories with online book sale data and argue that simultaneous search explains the data better than sequential search. Dinerstein et al. (2014) estimate online search costs and retail margins with a consumer search model based on the "consideration set" approach, and apply them to evaluate the effect of a search redesign by eBay in 2011. Although empirical analysis is beyond the scope of this paper, we think that our equilibrium model is tractable and structured enough to be taken to data.

3 Environment

The market consists of n sellers, each indexed by $i = \{1, ..., n\}$, and a unit mass of consumers. The sellers face no capacity constraint, while each consumer demands one unit among all products. The sellers simultaneously announce prices. Consumers observe those prices and sequentially search for the best product.

Each seller i supplies a product at no fixed cost and constant marginal cost c_i. We denote by $p_i \in \mathcal{R}_+$ seller i's price. In addition, we let $\mathbf{p}$ denote the price vector for all sellers (i.e., $\mathbf{p} = (p_1, ..., p_n)$) and $\mathbf{p}_{-i}$ denote the price vector except for seller i's price (i.e., $\mathbf{p}_{-i} = (p_1, ..., p_{i-1}, p_{i+1}, ..., p_n)$). Denote by $D_i(\mathbf{p})$ the measure of consumers who eventually purchase product i. Seller i's profit is then defined to be $\pi_i(\mathbf{p}) \equiv D_i(\mathbf{p})(p_i - c_i)$. Each seller maximizes his profit $\pi_i(\mathbf{p})$.

Each consumer's random utility for seller i's product is given by $\tilde{V}_i = V_i + Z_i$. The first component V_i is the (representative) consumer's prior value for product i, while the second component Z_i is the residual part that is revealed to the consumer only when she visits seller i and inspects his product. As for prices, we let $\mathbf{v} = (v_1, ..., v_n)$ and $\mathbf{z} = (z_1, ..., z_n)$ denote realized value profiles for each component.

The products are horizontally differentiated. For each consumer, V_i and Z_i are independently and identically drawn from the intervals $[\underline{v}_i, \overline{v}_i]$ and $[\underline{z}_i, \overline{z}_i]$ according to the distribution functions F_i and G_i, respectively. In addition, they are independent of each other and across the products. We allow each support to be infinite on both sides and assume that F_i and G_i have continuously differentiable density functions f_i and g_i, respectively. Independence across the products allows us to utilize the optimal search solution by Weitzman (1979), while independence between V_i and Z_i leads to a clean and easy-to-interpret characterization.[7]

Search is costly, but recall is costless. Specifically, each consumer must visit seller i and discover her match value z_i in order to be able to purchase product i. She incurs search costs $s_i (> 0)$ on her first visit. Then, she can purchase the product immediately or recall it any time later. Each consumer can leave the market and take an outside option at any point. The

[7] We note that independence between V_i and Z_i is restrictive not by itself, but because of a joint additive-utility specification ($\tilde{V}_i = V_i + Z_i$). It is always possible to reinterpret (redefine) Z_i, so that it is independent of V_i. In this case, a restriction is only due to the utility specification. On the other hand, Z_i can be redefined as $Z_i \equiv \tilde{V}_i - E[\tilde{V}_i|V_i]$. In this case, independence between V_i and Z_i imposes a restriction.

value of the outside option is given by u_0 for all consumers.

Each consumer's ex post utility depends on her total value for the purchased product $\tilde{v}_i$, its price p_i, and her search history. Let N be the set of sellers the consumer visits. If she purchases product i (in N), then her ex post utility is equal to

$$U(v_i, z_i, p_i, N) = v_i + z_i - p_i - \sum_{j \in N} s_j.$$

If she takes the outside option, then her ex post utility is equal to $U(N) = u_0 - \sum_{j \in N} s_j$. Each consumer is risk neutral and maximizes her expected utility.

The market proceeds as follows. First, the sellers simultaneously announce prices $\mathbf{p}$. Then, each consumer shops (searches) based on available information $(\mathbf{p}, \mathbf{v})$. We study subgame perfect Nash equilibria of this market game.[8] We first characterize consumers' optimal shopping behavior and then analyze the pricing game among the sellers.

4 Consumer Behavior

In this section, we analyze consumers' sequential search problems.

4.1 Optimal Shopping

Given prices $\mathbf{p}$ and prior values $\mathbf{v}$, each consumer faces a sequential search problem. She decides in which order to visit the sellers and, after each visit, whether to stop, in which case she chooses which product to purchase, if any, among those she has inspected so far, or visit another seller. Although this is a complex combinatorial problem in general, an elegant solution is available by Weitzman (1979). Independence between v_i and z_i leads to an even sharper characterization, as reported in the following proposition.[9]

Proposition 1 *Given* $\mathbf{p} = (p_1, ..., p_n)$ *and* $\mathbf{v} = (v_1, ..., v_n)$*, the (representative) consumer's*

[8] For notational simplicity, we do not formally define consumers' search strategies. See Weitzman (1979) for a formal (recursive) description of search strategy.

[9] The measure of consumers who are indifferent over multiple choices is negligible, because F_i and G_i are continuously increasing for all i. We omit a description of those consumers' behavior.

optimal search strategy is as follows: for each i, let z_i^* be the value such that

$$s_i = \int_{z_i^*}^{\overline{z}_i} (1 - G_i(z_i))dz_i. \tag{1}$$

(i) Search order: the consumer visits the sellers in the descending order of $v_i + z_i^* - p_i$ (i.e., she visits seller i before seller j if $v_i + z_i^* - p_i > v_j + z_j^* - p_j$).

(ii) Stopping: let N be the set of sellers the consumer has visited so far. She stops, and takes the best available option by the point, if and only if

$$\max\{u_0, \max_{i \in N} v_i + z_i - p_i\} > \max_{j \notin N} v_j + z_j^* - p_j.$$

Weitzman's solution is based on a single index for each option (seller). Let r_i be the value ("reservation utility") such that the consumer is indifferent between obtaining utility r_i immediately (which saves additional search costs s_i) and visiting seller i (which gives her an option to choose between r_i and $v_i + z_i - p_i$):

$$r_i = -s_i + \int_{\underline{z}_i}^{\overline{z}_i} \max\{r_i, v_i + z_i - p_i\} dG_i(z_i).$$

Weitzman (1979) shows that the optimal search strategy is to visit the sellers in the decreasing order of r_i and stop as soon as the best realized utility by the point exceeds all remaining r_i's.

In our model, due to the random additive-utility specification, $r_i = v_i + z_i^* - p_i$, where z_i^* is given by equation (1). Notice that z_i^* is independent of v_i and p_i but strictly decreasing in s_i, both of which are useful for the subsequent analysis. Intuitively, z_i^* represents the net option value of visiting seller i and learning z_i. Therefore, it inherits the independence properties of Z_i and decreases as the associated search costs s_i rise. We note that, whereas $z_i^* < \overline{z}_i$ for any $s_i > 0$, $z_i^* \geq \underline{z}_i$ if and only if s_i is below $E[Z_i] - \underline{z}_i$.

4.2 Shopping Outcomes

Despite the simplicity of Weitzman's solution, it is still difficult to summarize consumers' shopping outcomes and compute demand for each seller. Consider the simplest case where there are two sellers and no consumer outside option (i.e., $u_0 = -\infty$). Even in this case, there are three different paths through which a consumer eventually purchases product i. First, a consumer may visit seller i first and purchase immediately. Second, a consumer may visit seller i first, try seller j as well, but recall product i. Third, a consumer may visit seller j first but purchase product i. Total demand for seller i is the sum of all these demands. The main difficulty lies in that the number of purchase paths grows factorially fast as the number of sellers n increases.

Our main breakthrough is to identify a necessary and sufficient condition for consumers' eventual purchase decisions and, therefore, provide a simple way to summarize shopping outcomes. In order to motivate the result, consider the same duopoly case as above. The three purchase paths for product i correspond to each of the following conditions:

(i) $v_i + z_i^* - p_i > v_j + z_j^* - p_j$ (visit i first) and $v_i + z_i - p_i > v_j + z_j^* - p_j$ (stop at i).

(ii) $v_i + z_i^* - p_i > v_j + z_j^* - p_j$ (visit i first), $v_i + z_i - p_i < v_j + z_j^* - p_j$ (visit j as well), and $v_i + z_i - p_i > v_j + z_j - p_j$ (prefer i to j).

(iii) $v_i + z_i^* - p_i < v_j + z_j^* - p_j$ (visit j first), $v_i + z_i^* - p_i > v_j + z_j - p_j$ (visit i as well), and $v_i + z_i - p_i > v_j + z_j - p_j$ (prefer i to j).

Notice that the first condition can be simplified to $v_i + \min\{z_i, z_i^*\} - p_i > v_j + z_j^* - p_j$, while the second and the third conditions together can be reduced to $v_i + \min\{z_i, z_i^*\} - p_i \leq v_j + z_j^* - p_j$ and $v_i + \min\{z_i, z_i^*\} - p_i > v_j + z_j - p_j$. Intuitively, a consumer purchases product i if she either does not visit seller j or finds a sufficiently low realized value of z_j. Combining these inequalities, we arrive at the following single inequality:

$$v_i + \min\{z_i, z_i^*\} - p_i > v_j + \min\{z_j, z_j^*\} - p_j.$$

This simple condition can be extended for the general case by considering each pair of sellers and accommodating the outside option, as formally reported in the following theorem.

Theorem 1 (Eventual Purchase) *Let $w_i \equiv v_i + \min\{z_i, z_i^*\}$ for each i. Given $(\mathbf{p}, \mathbf{v}, \mathbf{z})$, the consumer purchases product i if and only if $w_i - p_i > u_0$ and $w_i - p_i > w_j - p_j$ for all $j \neq i$.*

Theorem 1 suggests that consumers' shopping outcomes can be summarized as in canonical discrete-choice models.[10] The only difference is that consumers' purchase decisions are

[10]Theorem 1 holds even if prices are not observable to consumers before search, as long as consumers have

made based, neither on true values $\tilde{v}_i$ nor on prior values v_i, but on newly identified values w_i, which we refer to as *effective values*. Clearly, w_i is related to underlying values $\tilde{v}_i$ and v_i. In particular, w_i converges to $\tilde{v}_i$ as s_i tends to 0 (in which case z_i^* approaches $\overline{z}_i$) and is determined only by v_i as s_i tends to infinity (in which case z_i^* approaches $-\infty$). Intuitively, if there are no search costs (i.e, $s_i = 0$ for all i), each consumer makes a fully informed decision and purchases the product that offers the largest net utility (i.e., $w_i = \tilde{v}_i$ for all i). To the contrary, if search costs grow arbitrarily large (i.e., $s_i \to \infty$ for all i), then consumers' purchase decisions depend only on prior values. In general, search frictions make consumers' hidden values $\mathbf{z}$ imperfectly reflected in their purchase decisions. The problem becomes more severe, and consumers rely less on $\mathbf{z}$, as search frictions increase.

The specific upper truncation structure of w_i derives from the monotonicity properties of Weitzman's solution. If a consumer visits seller i, that means that she has not found a product that gives her more than $v_i + z_i^* - p_i$ (optimal stopping rule) and Weitzman's indices for all remaining sellers are lower than $v_i + z_i^* - p_i$ (optimal search rule). Then, she stops and purchases product i with probability 1 if and only if $z_i \geq z_i^*$. This implies that each consumer's eventual purchase probability is independent of z_i conditional on $z_i > z_i^*$, which is equivalent to z_i's entering into consumers' purchase decisions only through $\min\{z_i, z_i^*\}$.

In order to utilize Theorem 1, we let H_i denote the distribution function for the new random variable $W_i = V_i + \min\{Z_i, z_i^*\}$, that is,

$$H_i(w_i) \equiv \int_{\underline{z}_i}^{z_i^*} F_i(w_i - z_i)dG_i(z_i) + \int_{z_i^*}^{\overline{z}_i} F_i(w_i - z_i^*)dG_i(z_i). \tag{2}$$

In addition, we let $\underline{w}_i \equiv \underline{v}_i + \min\{\underline{z}_i, z_i^*\}$ and $\overline{w}_i \equiv \overline{v}_i + z_i^*$, so that $[\underline{w}_i, \overline{w}_i]$ is the support of H_i. Notice that if s_i tends to 0, then z_i^* converges to $\overline{z}_i$ and, therefore, H_i becomes the convolution of F_i and G_i. If s_i explodes, then z_i^* approaches negative infinity, in which case H_i depends only on F_i. A closed-form solution for $H_i(w_i)$ is available for some commonly used distributions. See the online appendix for three examples (Section A).

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correct beliefs about prices (i.e., in equilibrium). However, the result does not hold if a seller deviates, because consumers' search decisions are based on their expectations about prices, while their final purchase decisions depend on actual prices charged. Still, the condition can be modified in a straightforward fashion and used to study the model with unobservable prices. See Section 6.2.

5 Market Equilibrium

Theorem 1 implies that the demand function for each seller can be derived as in standard discrete-choice models. Each consumer purchases product i if and only if $w_i - p_i$ exceeds u_0 and $w_j - p_j$ for all $j \neq i$. Let X_i denote each consumer's (random) utility from the best alternative to product i and $\tilde{H}_i$ denote its distribution, that is, $X_i \equiv \max\{u_0, \max_{j \neq i} W_j - p_j\}$ and $\tilde{H}_i(x_i) \equiv Pr\{X_i \leq x_i\}$. Then, aggregate demand for product i is equal to

$$D_i(\mathbf{p}) = \int (1 - H_i(x_i + p_i)) d\tilde{H}_i(x_i).$$

Therefore, a necessary (first-order) condition for equilibrium p_i is given by

$$\frac{1}{p_i - c_i} = -\frac{dD_i(\mathbf{p})/dp_i}{D_i(\mathbf{p})} = \frac{\int h_i(x_i + p_i) d\tilde{H}_i(x_i)}{\int (1 - H_i(x_i + p_i)) d\tilde{H}_i(x_i)}. \tag{3}$$

This is a standard optimal pricing formula: $-dD_i(\mathbf{p})/dp_i$ is the measure of marginal consumers who are indifferent between product i and the best alternative (i.e., $w_i - p_i = x_i$). The formula states that the optimal markup is inversely proportional to the proportion of marginal consumers among those who purchase product i.

The demand system above exhibits standard properties for imperfect substitutes: demand for seller i decreases in own price p_i and increases in competitors' prices $\mathbf{p}_{-i}$. However, it does not behave well in general: depending on the precise shapes of H_i and $\tilde{H}_i$, the right-hand side in equation (3) may not be monotone in $\mathbf{p}$, in which case there may exist no or multiple pure-strategy equilibria. Our discrete-choice reformulation, however, allows us to borrow existing results on discrete-choice models, rather than developing our own specific results. This is particularly useful for equilibrium existence and uniqueness, for which various general results are available.

We establish the existence and uniqueness of market equilibrium by building upon Quint (2014), who provides one of the most general characterizations for standard discrete-choice models. His result makes use of the following two assumptions:

Assumption 1 *For each i, both H_i and $1 - H_i$ are log-concave.*

Assumption 2 *For each i, the support of H_i has no upper bound (i.e., $\overline{w}_i = \infty$).*

The following result is a special version of his Theorem 1 and Lemma 1.

Theorem 2 (Quint, 2014) *Under Assumption 1, $D_i(\mathbf{p})$ is log-concave in p_i, and $\log D_i(\mathbf{p})$ has strictly increasing differences in p_i and p_j. Under Assumptions 1 and 2, there exists a unique equilibrium, which is in pure strategies, in the pricing game among the sellers.*

Intuitively, log-concavity of $1 - H_i$ (i.e., $h_i/(1 - H_i)$ increasing) generates the effect of pushing up the right-hand side in equation (3) as p_i rises. Log-concavity of $\tilde{H}_i$ (which stems from log-concavity of H_j's) reinforces the basic effect, thereby ensuring not only that seller i's best response to $\mathbf{p}_{-i}$ is unique, but also that the pricing game is a supermodular game. Assumption 2 guarantees that each seller's demand is positive at any price and, therefore, each seller always chooses a price above c_i. The existence of a pure-strategy equilibrium follows as an application of more general existence theorems for supermodular games (see, e.g., Vives, 2005). Its uniqueness is not implied by general theory but can be obtained by exploiting the linear structure of the model, namely that $D_i(\mathbf{p})$ is invariant when all prices, together with $-u_0$, increase by the same amount.

Unlike in standard discrete-choice models where H_i's are primitives, they depend on F_i, G_i, and s_i in a specific way in our model, as shown in equation (2). Indeed, Assumption 1 can fail even with a strong assumption that both f_i and g_i are log-concave.[11] In order to understand the underlying problem, consider the case where F_i is degenerate at v_i. In this case, $H_i(w_i)$ jumps up at $v_i + z_i^*$ (see the solid line, corresponding to $\alpha = 0$, in the left panel of Figure 1) and, therefore, cannot be globally log-concave. When F_i is not degenerate, the atom at $v_i + z_i^*$ is continuously scattered, in which case H_i is continuous. However, if F_i is sufficiently concentrated around v_i, then the slope of H_i at $v_i + z_i^*$ can be arbitrarily large (see the dashed line, corresponding to $\alpha = 0.1$, in the left panel of Figure 1). Therefore, H_i may still fail to be log-concave.

Based on this observation, we provide a set of sufficient conditions under which Theorem 2 applies to our model.

Proposition 2 *Suppose that both f_i and g_i are log-concave and the support of F_i has no upper bound (which is necessary and sufficient for Assumption 2).*

[11] If the density function f is log-concave, then both distribution function F and survival function $1 - F$ are log-concave by Prékopa's Theorem. See Bagnoli and Bergstrom (2005) for more details.

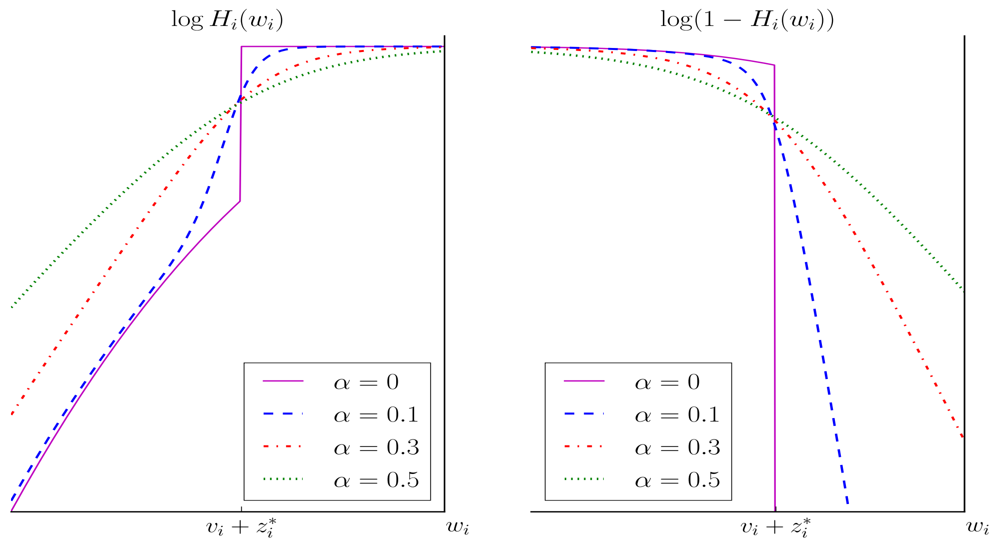

Figure 1: $log(H_i(w_i))$ and $log(1 - H_i(w_i))$ for different dispersion levels of F_i. For both panels, $F_i(v_i) = 1/(1 + e^{-v_i/\alpha})$ (logistic), $G_i(z_i) = \Phi(z_i)$ (standard normal), and $s_i = 1$.

- *The survival function $1 - H_i$ is always log-concave.*
- *If the variance of V_i is sufficiently large and F_i has no lower bound (i.e., $\underline{v}_i = -\infty$) or $f_i(\underline{v}_i) = 0$, then H_i is also log-concave.*[12]
- *If $f_i'(u_0 + c_i - z_i^*) \leq 0$, then H_i is log-concave above $w_i > \max\{u_0 + c_i, \underline{w}_i\}$.*

The first result states that unlike H_i, log-concavity of f_i and g_i suffices for log-concavity of $1 - H_i$. The difference lies in the fact that, as shown in the right panel of Figure 1, when F_i is degenerate at v_i, $1 - H_i$ jumps down at the discontinuity point $v_i + z_i^*$, which does not disrupt log-concavity of the function. Log-concavity of f_i and g_i also guarantees log-concavity of H_i in the limit as s_i tends to 0 (in which case $z_i^* \to \overline{z}_i$) or ∞ (in which case $z_i^* \to -\infty$), but more restrictions are required when $s_i \in (0, \infty)$. The second result is based on the idea that if F_i is sufficiently spread out, then the atom of $\min\{Z_i, z^*\}$ at z^* is so well scattered out through the support that H_i can be log-concave. The requirement on $\underline{v}_i$ ensures that h_i does not jump at $w_i = \underline{v}_i + z_i^*$. In the left panel of Figure 1, if α (which measures

[12]To be precise, we consider $V_i^\sigma = \sigma V_i$ for a given V_i and show that the effective value distribution H_i^σ corresponding to V_i^σ is log-concave if σ is sufficiently large.

dispersion of F_i) is sufficiently large, then H_i increases sufficiently slowly around $v_i + z_i^*$ and, therefore, becomes log-concave.[13] The final result is based on the observation that Theorem 2 applies as long as Assumption 1 holds for a relevant parameter region. Since each seller always chooses $p_i \geq c_i$, consumers such that $w_i \leq u_0 + c_i$ are effectively irrelevant to seller i's optimal pricing, because $w_i - p_i \leq w_i - c_i \leq u_0$ and, therefore, those consumers never purchase from the seller. A simple condition $f_i'(u_0 + c_i - z_i^*) \leq 0$ ensures that H_i is log-concave for all relevant consumers.[14]

Assumptions 1 and 2 are sufficient but not necessary for the existence of a pure-strategy equilibrium. Therefore, their violation does not imply that there does not exist a pure-strategy equilibrium. Although it is beyond the scope of this paper to provide a more general sufficient condition, it is easy to see that in the symmetric environment (formally defined in the next section) with degenerate F_i, there typically does not exist a symmetric pure-strategy equilibrium. On the one hand, each seller has an incentive to undercut the opponents, because it induces all consumers to visit the seller first and, therefore, yields a discrete jump to the seller's profit. On the other hand, product differentiation implies that zero markup (i.e., $p_i = c_i$) cannot be an equilibrium outcome either. This indicates that there can exist only a mixed-strategy equilibrium, whose characterization is surprisingly demanding even for the uniform-distribution case. In the online appendix (Section C), we construct an equilibrium for the case where G_i is an exponential distribution, exploiting its unique features.

6 Comparative Statics

In this section, we present a set of comparative statics results regarding preference diversity and search frictions.[15] For clear insights as well as tractability, we restrict attention to the

[13] Haan et al. (2017) conjecture this result and provide a set of confirming numerical examples. Our result formalizes their conjecture. In the example used in Figure 1, numerically, H_i is log-concave when $\alpha \geq 0.281$. If $F_i = \mathcal{N}(0, \alpha^2)$, instead, then H_i is log-concave when $\alpha \geq 0.621$.

[14] This condition always holds with monotone density functions (e.g., exponential and half-normal) and is trivial to check with most distribution functions. In addition, if the support of F_i has no upper bound (as implied by Assumption 2), then it must eventually decrease. Therefore, the condition necessarily holds if $u_0 + c_i$ is sufficiently large. See the online appendix (Section A) for some straightforward examples. We thank an anonymous referee for suggesting this simple and useful condition.

[15] Given our discrete-choice reformulation, most standard results immediately follow from the existing literature. See Quint (2014) for several basic results. One important question in the literature is the effect of more intense competition (i.e., the number of sellers n) on market prices. For example, Chen and Riordan (2008)

case where the sellers are symmetric. Precisely, we assume that for all i, $F_i = F$, $G_i = G$, $c_i = c$, $s_i = s$, and $H_i = H$. We also maintain Assumptions 1 and 2,[16] so that there is a unique equilibrium, which is in symmetric pure strategies, and $D_i(\mathbf{p})$ is well-behaved. We let p^* and $\pi(p^*)$ denote the symmetric equilibrium price and profit, respectively.

6.1 Preference Diversity

We begin by establishing a result that relates dispersion of H to p^*. The result not only is useful for our subsequent comparative statics results, but also has a direct contribution to the general discrete-choice literature. Product differentiation provides a way to overcome the Bertrand paradox: each seller has some loyal consumers (who value the seller's product more than other products) and, therefore, can set a positive markup even under Bertrand competition. Given this observation, it is plausible that the more differentiated consumers' preferences are, the higher prices the sellers charge. A challenge has been to identify an appropriate measure of product differentiation (preference diversity). In their seminal work, Perloff and Salop (1985) show that constant scaling of consumers' preferences necessarily increases market prices, but find that the result does not extend for mean-preserving spreads. Our result offers an answer to this question.

We utilize the following stochastic order, typically referred to as *dispersive order*.[17]

Definition 1 *The distribution function H_2 is more dispersed than the distribution function H_1 if $H_2^{-1}(b) - H_2^{-1}(a) \geq H_1^{-1}(b) - H_1^{-1}(a)$ for any $0 < a \leq b < 1$. Likewise, H_2 is more dispersed than H_1 above w if $H_2^{-1}(b) - H_2^{-1}(a) \geq H_1^{-1}(b) - H_1^{-1}(a)$ for any $H_1(w) < a \leq b < 1$.*

Intuitively, a more dispersed distribution has more spread-out density and, therefore, increases more slowly, which is equivalent to its quantile function H^{-1} increasing faster.[18]

provide a condition under which the symmetric duopoly price is higher than the monopoly price, and Gabaix et al. (2016) analyze asymptotic price behavior when there are numerous sellers (i.e., as n tends to infinity).

[16] To be precise, we require H to be log-concave only above $u_0 + c$. Note that, alternatively, one can directly assume that the symmetric equilibrium price is characterized by the first-order condition and $D_i(\mathbf{p})$ satisfies the properties in Theorem 2.

[17] Note that the definition requires that the inequality should hold for all interior quantiles but imposes no restriction on the boundary points. This fact is used in our proofs of Propositions 4 and 5.

[18] Dispersive order is location-free and, therefore, neither is implied by nor implies second-order stochastic dominance. Mean-preserving dispersive order, however, implies mean-preserving spread: if H_2 is more dis-

Dispersive order is useful in various problems (see Section 2) but also has some limitations. In particular, two random variables with a common finite support can never be ranked in terms of dispersive order.[19] This problem, however, does not arise in our model because of Assumption 2. The second definition with qualifier "above w" is useful in our model, because the left tail of H does not affect p^*, as explained for the last result of Proposition 2.

Proposition 3 *In the symmetric environment, the equilibrium price p^* increases as H becomes more dispersed above $u_0 + c$ and $H(u_0 + c)$ weakly decreases.*

Proof. In the symmetric environment, equation (3) reduces to

$$\frac{1}{p^* - c} = \frac{\int_{\underline{w}}^{\infty} h(\max\{u_0 + p^*, w\}) dH(w)^{n-1}}{\int_{\underline{w}}^{\infty} (1 - H(\max\{u_0 + p^*, w\})) dH(w)^{n-1}}. \tag{4}$$

The left-hand side strictly decreases in p^*, while the right-hand side increases in p^* (see the appendix). Therefore, it suffices to prove that given p^*, the right-hand side falls when H changes as indicated. Let $\phi \equiv H(u_0 + p^*)$ and change the variable with $a = H(w)$. Then,

$$\frac{1}{p^* - c} = \frac{h(H^{-1}(\phi))\phi^{n-1} + \int_{\phi}^{1} h(H^{-1}(a)) da^{n-1}}{\frac{1}{n}(1 - \phi^n)}.$$

If H becomes more dispersed above $u_0 + c$, then $dH^{-1}(a)/da = 1/h(H^{-1}(a))$ increases for each $a \geq \phi$, which lowers the right-hand side. If, in addition, $H(u_0 + c)$ decreases, then ϕ also decreases, because $p^* > c$ and a distribution function crosses a less dispersed one only once from above. This further lowers the right-hand side, because it has the same effect as lowering p^*, which enters only through $\phi = H(u_0 + p^*)$, in the right-hand side. ■

The relationship between dispersion of H and p^* is particularly clear when there is no outside option (i.e., $u_0 = -\infty$). In that case, the second condition regarding $H(u_0 + c)$ is vacuous and p^* depends only on the dispersive order of H. In order to understand this result,

persed than H_1 with the same mean, then H_2 is a mean-preserving spread of H_1. See Shaked and Shanthikumar (2007) for further details.

[19] Suppose H_1 and H_2 have the same bounded support $[\underline{w}, \overline{w}]$. By definition,

$$H_1^{-1}(1) - H_1^{-1}(0) = H_2^{-1}(1) - H_2^{-1}(0) = \underline{w} - \overline{w}.$$

But then, $H_2^{-1}(b) - H_2^{-1}(a) \geq H_1^{-1}(b) - H_1^{-1}(a)$ for any $0 < a \leq b < 1$ if and only if $H_1 = H_2$.

notice that in the symmetric equilibrium with no outside option, each consumer purchases product i if and only if her effective value for product i exceeds those for the other products. This means that $D_i(\mathbf{p})$ is identical to the probability that W_i is the first-order statistic among all W_j's, and $-dD_i(\mathbf{p})/dp_i$ is identical to the probability density that W_i coincides with the second-order statistic $\max_{j \neq i} W_j$. As H becomes more dispersed, the difference between the first- and the second-order statistics grows larger and, therefore, the density of the event $\{W_i = \max_{j \neq i} W_j\}$ decreases. This implies fewer marginal consumers and, therefore, higher market prices.[20]

In general, dispersive order alone does not suffice for the result. This is because dispersive order is concerned only with the slope of the quantile function and, therefore, location-free. For example, $\mathcal{N}(\mu_2, \sigma_2^2)$ is more dispersed than $\mathcal{N}(\mu_1, \sigma_1^2)$ if and only if $\sigma_2 > \sigma_1$, independent of μ_1 and μ_2. When there is no outside option, each consumer must purchase one of the products, even if she receives arbitrarily low negative utility. In that case, the location of H does not affect demand and, therefore, p^* is determined only by dispersion of H. When there is a finite outside option, the location clearly matters: if H shifts to the left, then more consumers would opt to take the outside option, which provides an incentive for the sellers to lower their prices. The second requirement in Proposition 3 ensures that H does not shift to the left so much that this location effect does not disrupt the dispersion effect. Naturally, a sufficient condition for the second requirement is that H increases in the sense of first-order stochastic dominance (i.e., H shifts to the right).

In our model of consumer search, H is determined by the two primitive distributions F and G. The following proposition illustrates how dispersion of each distribution translates into dispersion of H and, therefore, affects market prices (combined with Proposition 3).

Proposition 4 *In the symmetric environment, H becomes more dispersed above $u_0 + c$*

- *as G becomes more dispersed, provided that f is log-concave, or*
- *as F becomes more dispersed, provided that g is log-concave, F has decreasing density above $u_0 + c - z^*$ and $\underline{v} \in (-\infty, u_0 + c - z^*]$.*

Proof. We use the following mathematical result: $X \equiv X_1 + X_2$ becomes more dispersed whenever X_2 becomes more dispersed, if and only if X_1 has log-concave density (Theorem

[20]This explanation based on the relationship between the first- and the second-order statistics is borrowed from Zhou (2017). See also Gabaix et al. (2016).

3.B.8 in Shaked and Shanthikumar, 2007). The first result directly follows from the fact that if Z becomes more dispersed, then $\min\{Z, z^*\}$ also becomes more dispersed (shown in the appendix). A similar proof does not apply to dispersion of F, because $\min\{Z, z^*\}$ has a mass on z^* and, therefore, never has log-concave density. We provide a direct proof for the second result in the appendix. ■

An increase in dispersion of G tends to increase the value of search, because each consumer always has an option not to take an inspected product and, therefore, her ex post search payoff is convex. Consumers would then visit more sellers and acquire more information, which makes their effective values more dispersed. This result is in stark contrast to its counterpart with unobservable prices: Anderson and Renault (1999) find that the equilibrium price initially decreases but eventually increases as the distribution of hidden values (G) gets scaled up. This difference is precisely due to the difference in price observability: in the same environment as Anderson and Renault (1999) (with a degenerate F and no outside option), if G is degenerate, then the equilibrium price is minimized in our model, because the pricing game reduces to standard Bertrand competition, while it is maximized in their model, because of the Diamond paradox.

Notice that the two conditions in Proposition 4 are not symmetric. Since $\min\{Z, z^*\}$ cannot have log-concave density (unless $s = 0$), the mathematical result in the proof implies that H does not necessarily become more dispersed when F becomes more dispersed, no matter what regularity assumption is imposed on G. Economically, this is because, whereas more ex ante preference diversity directly increases ex post preference diversity, it may also deter more consumers from searching and learning about their total values, which reduces preference diversity. Proposition 4 shows that a certain (rather strong) distributional assumption on F, together with log-concavity of g, can still ensure that the former (direct) effect dominates the latter effect.

6.2 Search Costs

It is well-known that if prices are unobservable before search, then market prices tend to rise with search costs.[21] Intuitively, an increase in search costs reduces the value of additional

[21] To be precise, the result depends on the shapes of the relevant distributions. For example, Anderson and Renault (1999) consider the case where F is degenerate and show that the symmetric equilibrium price p^*

search, which strengthens each seller's market power over visiting consumers and, therefore, leads to higher prices. The following result shows that the assumption of observable prices reverses the result under a standard regularity assumption.

Proposition 5 *In the symmetric environment, both equilibrium price p^* and equilibrium profit $\pi(p^*)$ decrease in s, provided that f is log-concave.*

Proof. We prove the price result here and relegate a proof for the profit result to the appendix. For the price result, we show that as s increases, H becomes less dispersed and $H(u_0 + c)$ increases. The result then follows by applying Proposition 3. If s increases, then z^* falls (see equation (1)). This implies that $W = V + \min\{Z, z^*\}$ decreases in the sense of first-order stochastic dominance and, therefore, $H(u_0 + c)$ rises. A decrease of z^* also reduces dispersion of $\min\{Z, z^*\}$: since the distribution function for $\min\{Z, z^*\}$ coincides with $G(z)$ until z^* and then jumps to 1, the corresponding quantile function is given by $\min\{G^{-1}(a), z^*\}$, which becomes weakly flatter at any $a \in (0, 1)$ as z^* decreases. The desired result then follows by letting $X_1 = V$ and $X_2 = \min\{Z, z^*\}$ and applying the mathematical result in the proof of Proposition 4 above. ■

Intuitively, observable prices directly influence consumer search: the lower price a seller offers, the more consumers visit him. An increase in search costs raises the value of attracting consumers, because they search less and are more likely to purchase from their visit. This intensifies price competition among the sellers and leads to lower prices. Another way to understand the result (in fact, the idea used in the proof above) is through the relationship between search costs and the distribution of consumers' effective values. An increase in s induces consumers to search less. They stop with lower values (thus, H decreases in the sense of first-order stochastic dominance) and earlier (thus, H becomes less dispersed), both of which provide an incentive for the sellers to lower their prices.

In the duopoly model with no outside option, Haan et al. (2017) identify two opposing effects, a non-directed search effect (exploiting visiting consumers further) and a search direction effect (more eager to attract consumers), and establish the same result by showing

increases in s if $1 - G$ is log-concave but decreases in s if $1 - G$ is log-convex, assuming that there exists a symmetric pure-strategy market equilibrium in both cases. Haan et al. (2017) provide some sufficient conditions under which p^* rises with s in the duopoly environment with non-degenerate F. In the online appendix (Section D), we provide an example in which F is non-degenerate, $1 - G$ is log-concave, but p^* decreases in s.

that the latter necessarily dominates the former under some regularity assumptions. Their analysis is based on an explicit derivation of demand from each purchase path, which is unmanageable in the general oligopoly environment. Proposition 5 complements their analysis, not only by showing that their result goes far beyond their simple environment but also by providing an alternative approach based on our discrete-choice reformulation (Theorem 1) and new general comparative statics result (Proposition 3).

In order to better understand the difference between observable and unobservable prices, let p_i denote the actual price seller i charges and p_i^e denote the price consumers expect seller i to charge. Assuming that all other sellers advertise and charge p^* and applying the same logic as for Theorem 1, a consumer purchases product i if and only if

$$\min\{v_i + z^* - p_i^e, v_i + z_i - p_i\} > \max\{u_0, \max_{j \neq i} w_j - p^*\}.$$

For expositional clarity, suppose that there is no outside option and let $\tilde{W}_i \equiv \max_{j\neq i} W_j$. Then, demand for seller i is given by

$$D_i(p_i, p_i^e, p^*) = E\left[\int_{\tilde{W}_i - p^* - z^* + p_i^e}^{\infty} (1 - G(\tilde{W}_i - p^* - v_i + p_i))dF(v_i)\right]. \tag{5}$$

If prices are unobservable, then in the symmetric equilibrium,

$$-\frac{dD_i(p_i, p_i^e, p^*)}{dp_i}|_{p_i = p_i^e = p^*} = E\left[\int_{\tilde{W}_i - z^*}^{\infty} g(\tilde{W}_i - v_i)dF(v_i)\right],$$

while if prices are observable (in which case $p_i = p_i^e$), then

$$-\frac{dD_i(p_i, p_i, p^*)}{dp_i}|_{p_i = p^*} = E\left[\int_{\tilde{W}_i - z^*}^{\infty} g(\tilde{W}_i - v_i)dF(v_i) + (1 - G(z^*))f(\tilde{W}_i - z^*)\right].$$

In the second equation, the first term represents consumers on the *intensive* margin, who are indifferent between purchasing product i and the best alternative (i.e., $V_i + Z_i = \tilde{W}_i$) among those who visit seller i (i.e., $V_i + z^* \geq \tilde{W}_i$), while the second term captures consumers on the *extensive* margin, who are indifferent between visiting seller i and the best alternative (i.e., $V_i + z^* = \tilde{W}_i$) but will purchase product i if they visit seller i (i.e., $V_i + Z_i \geq \tilde{W}_i$). As shown above, the union of these two groups of consumers coincides with the event $\{W_i = \tilde{W}_i\}$.

When prices are unobservable, only the intensive margin is operative and, therefore, each seller faces fewer marginal consumers. It immediately follows that market prices are higher when they are unobservable before search than when they are observable.

The difference in the extensive margin is also responsible for different comparative statics results regarding search costs. With observable prices, $-dD_i/dp_i$ coincides with the event $\{W_i = \tilde{W}_i\}$, leading to $-dD_i/dp_i = \int h(w)dH(w)^{n-1}$. It then follows that the price effect of search costs can be inferred from its clear effect on dispersion of H. With unobservable prices, this is no longer the case, because the absence of the extensive margin breaks the equality between $-dD_i/dp_i$ and the event $\{W_i = \tilde{W}_i\}$. In fact, the result is ambiguous even under certain regularity assumptions. An increase in s lowers z^* and, therefore, reduces the measure of marginal consumers given a realization of $\tilde{W}_i$ (the integral term in the expectation operator), which pushes up market prices. Intuitively, an increase in $s_i(= s)$ makes only the consumers who particularly value product i continue to visit seller i, which provides an incentive for the seller to raise her price. However, the distribution of $\tilde{W}_i$ (i.e., the expectation operator) itself changes with s: it falls in the sense of first-order stochastic dominance and becomes less dispersed as s rises. Depending on the shapes of F and G, this effect can increase or decrease the measure of marginal consumers and may even dominate the first effect. This makes the overall effect ambiguous (see footnote 21).

Proposition 5 opens up an interesting possibility that consumer surplus may increase with search costs.[22] An increase in search costs has a direct negative effect on consumer surplus. However, if the sellers lower their prices drastically in response, then overall consumer surplus may rise. In the online appendix (Section E), we provide an example in which this is indeed the case. It arises when the outside option is sufficiently unfavorable and there are sufficiently few sellers. In that case, each seller possesses strong market power and, therefore, charges a sufficiently high price. An increase in search costs induces them to drop their prices quickly, up to the point where the indirect effect outweighs the direct effect and, therefore, consumer surplus increases.

[22] Our discrete-choice reformulation simplifies the calculation of consumer surplus as well. See the online appendix (Section F).

6.3 Pre-search Information Quality

In our model, consumers search because they have imprecise information about their values for the products. This means that search frictions can also be measured by the extent to which consumers are uncertain about their total values. We now examine the effect of improving pre-search information quality on the equilibrium price p^*.

For tractability and clarity, we restrict attention to a Gaussian learning environment where both F and G are given by normal distributions with mean 0.[23] Furthermore, we assume that F has variance α^2, while G has variance $1-\alpha^2$, for some $\alpha \in (0,1)$ (i.e., $V \sim \mathcal{N}(0,\alpha^2)$ and $Z \sim \mathcal{N}(0,1-\alpha^2)$). The variances are deliberately chosen so as to ensure that $\tilde{V} = V + Z \sim \mathcal{N}(0,1)$ for any α, that is, the distribution for consumers' ex post (total) values is independent of α. The parameter α measures the quality of pre-search information: as α increases, consumers' ex post values $\tilde{V} = V + Z$ are influenced more by V and less by Z. We also assume that consumers have no outside option.

We find that, unlike in Propositions 4 and 5, the equilibrium price p^* (and, therefore, equilibrium profit $\pi(p^*) = (p^* - c)/n$ as well) may or may not increase as pre-search information quality improves. In particular, if the number of sellers is sufficiently large, then the equilibrium price p^* increases in α.[24]

Proposition 6 *In the symmetric environment where $V \sim \mathcal{N}(0,\alpha^2)$, $Z \sim \mathcal{N}(0,1-\alpha^2)$, and $u_0 = -\infty$, for each α that satisfies Assumption 1,[25] there exists an integer $n^*(\alpha)$ such that p^* increases in α if and only if $n \geq n^*(\alpha)$.*

Recall that market prices increase when H becomes more dispersed (Proposition 3). Importantly, the result depends only on H, not separately on F and G. This suggests that an increase in α has two opposing effects on p^*. On the one hand, it compresses G, which, as shown in Proposition 4, tends to decrease p^*. On the other hand, it spreads out F, which tends to increase dispersion of H and, therefore, drive up p^*.

[23] This Gaussian environment corresponds to the web search model in Choi and Smith (2016), who provide several comparative statics results for optimal sequential search behavior.

[24] This result is similar to a result on the revenue effect of disclosing more information in auctions. Specifically, Ganuza and Penalva (2010) show that providing more information, in the sense of dispersive order of conditional expectations, always lowers the seller's revenue if there are only two bidders but increases the seller's revenue whenever there are sufficiently many sellers. Despite several modeling differences, both results suggest that the intensity of competition is an important determinant for the effects of information provision.

[25] For example, if $s = 0.2$, then Assumption 1 holds (i.e., H is log-concave) when $\alpha \geq 0.313$.

In order to understand why the result depends on n, notice that, due to the specific truncation structure of $W = V + \min\{Z, z^*\}$, the relative impact of F on H grows larger as w increases. Meanwhile, consumers' maximal effective values are higher when there are more sellers. Together, these imply that the measure of marginal consumers becomes more sensitive to the behavior of F as n increases. Since F becomes more dispersed, while G becomes less dispersed, in α, p^* increases in α if and only if n is sufficiently large.

7 Conclusion

We analyze an oligopoly model in which each consumer sequentially searches for the best product based on partial product information and advertised (i.e., fully observable) prices. Unlike in some recent related studies, we do not restrict attention to the symmetric duopoly environment: we allow for any number of sellers and seller heterogeneity. This generalization is due to our eventual purchase theorem, which provides a simple static condition that fully summarizes consumers' sequential search outcomes and allows us to reformulate the pricing game among the sellers as a familiar discrete-choice problem. Exploiting this reformulation, we provide sufficient conditions under which there exists a unique market equilibrium, which is in pure strategies. In addition, by developing and utilizing a new general comparative statics result for discrete-choice models, we obtain some new and more general comparative statics results regarding preference diversity and search frictions.

Many interesting questions remain open. To name a few, we assume that all sellers are fully committed to their advertised prices. However, hidden fees, in various forms, are prevalent in reality. How does their potential presence affect consumer behavior and sellers' pricing incentives? We consider the case where each seller sells only one product, but it is the exception rather than the rule. How should a multi-product seller price and/or position his products? Should the seller choose an identical price, or introduce difference prices, for ex ante symmetric products? If the products are asymmetric, which product should the seller make prominent and how? Finally, we assume that pre-search information quality is exogenously given. However, more information can be provided by a seller's own advertisement or by a platform provider's market design. How much pre-search information are they willing to provide? Does competition necessarily induce the sellers to reveal more information?[26]

[26]See Ellison (2005) and Anderson and Renault (2006) for some important contributions and Armstrong

Our theoretical analysis has some broad implications for empirical research. Simultaneous search (à la Stigler, 1961; Chade and Smith, 2006) models situations in which each inspection takes a significant amount of time but the decision must be made within a certain time frame (for example, college admissions). It is not appropriate for many consumer search problems, because inspection is often straightforward for consumer products. However, it is believed to be more tractable than sequential search and, therefore, has been adopted by various empirical studies (see, e.g., Moraga-González et al., 2015; Pires, 2015). Our eventual purchase theorem greatly improves the tractability of sequential search, thereby significantly improving its applicability. In addition, it implies that existing theoretical and econometric results on discrete-choice models can be used to analyze consumer search markets. We think that this opens up numerous research opportunities, including econometric identification problems (e.g., what distortions arise if one estimates a discrete-choice model ignoring search frictions?) and revisiting previous empirical studies (e.g., comparing simultaneous search with sequential search).

Appendix: Omitted Proofs

Proof of Theorem 1. Sufficiency: $w_i - p_i > u_0$ implies that the consumer never takes an outside option u_0, because she is willing to visit at least one seller ($v_i + z_i^* - p_i > u_0$) and make a purchase ($v_i + z_i - p_i > u_0$). Given this, it suffices to show that if $w_i - p_i > w_j - p_j$, then the consumer never purchases product j.

- Suppose $z_j^* \leq z_j$, which implies that $w_j = v_j + z_j^*$. The consumer visits seller j only after seller i because $v_i + z_i^* - p_i \geq w_i - p_i > v_j + z_j^* - p_j$. However, once she visits seller i, she has no incentive to visit seller j because $v_i + z_i - p_i > v_j + z_j^* - p_j$.
- Suppose $z_j^* > z_j$, which implies that $w_j = v_j + z_j$. In this case, even if she visits seller j, she either recalls a previous product ($v_i + z_i - p_i > v_j + z_j - p_j$) or continues to search ($v_i + z_i^* - p_i > v_j + z_j - p_j$) and finds a better product ($v_i + z_i - p_i > v_j + z_j - p_j$).

Necessity: if $w_i - p_i < u_0$, then the consumer does not visit seller i ($v_i + z_i^* - p_i < u_0$) or does not purchase product i even if she visits seller i ($v_i + z_i - p_i < u_0$). If $w_i - p_i < w_j - p_j$

(2016) and the references therein for some recent developments.

for some $j \neq i$, then, for the same logic as above, the consumer never purchases product i. ■

Proof of Proposition 2.

(1) The survival function $1 - H_i$ is log-concave.

Equation (2) can be rewritten as

$$1 - H_i(w_i) = \int_{\underline{z}_i}^{\overline{z}_i} (1 - F_i(w_i - \min\{z_i, z_i^*\})g_i(z_i)dz_i. \tag{6}$$

Since $-w_i + \min\{z_i, z_i^*\}$ is concave in (w_i, z_i) and $1 - F_i(-x)$ is increasing and log-concave (due to log-concavity of $1 - F_i(x)$) in x, the composite function $1 - F_i(-(-w_i + \min\{z_i, z_i^*\}))$ is log-concave in (w_i, z_i). Since $g_i(z_i)$ is also log-concave and log-concavity is preserved under multiplication, the product $(1 - F_i(w_i - \min\{z_i, z_i^*\})g_i(z_i)$ is log-concave in (w_i, z_i). The desired result then follows from Prékopa's theorem, which states that if the integrand is log-concave, then the integral is also log-concave (see, e.g., Caplin and Nalebuff, 1991; Choi and Smith, 2016, for a formal statement of the theorem and its uses in related contexts).

(2) If the variance of V_i is sufficiently large, then H_i is also log-concave.

We provide a basic argument here, relegating a complete (long) proof to the online appendix (Section B). Given V_i and Z_i, let $V_i^\sigma \equiv \sigma V_i$ and $W_i^\sigma \equiv V_i^\sigma + \min\{Z_i, z_i^*\}$. In addition, let F_i^σ and H_i^σ denote the distribution functions for V_i^σ and W_i^σ, respectively. By the definition of V_i^σ,

$$F_i^\sigma(v_i^\sigma) = F_i\left(\frac{v_i^\sigma}{\sigma}\right), \; f_i^\sigma(v_i^\sigma) = \frac{f_i(v_i^\sigma/\sigma)}{\sigma}, \text{ and } (f_i^\sigma)'(v_i^\sigma) = \frac{f_i'(v_i^\sigma/\sigma)}{\sigma^2}.$$

Fixing $F_i^\sigma(v_i^\sigma) = r$ at a quantile, or equivalently $v_i^\sigma = \sigma F_i^{-1}(r)$, as σ explodes, f_i^σ converges to 0 at rate $1/\sigma$, and $(f_i^\sigma)'$ does at rate $1/\sigma^2$. Intuitively, as F_i^σ becomes more spread out, its density f_i^σ falls to 0 and the slope of the density converges to 0 even faster. Exploiting these differences in the converge rates, it follows that given $r \in [0, 1]$, as σ tends to infinity,

$$H_i^\sigma(w_i^\sigma) \to F_i(v_i), \; \sigma h_i^\sigma(w_i^\sigma) \to f_i(v_i), \text{ and } \sigma^2 (h_i^\sigma)'(w_i^\sigma) \to [1 - G_i(z_i^*)]f_i'(v_i)$$

where $w_i^\sigma \equiv (F_i^\sigma)^{-1}(r) + z_i^*$ and $v_i \equiv F_i^{-1}(r)$ (see the online appendix for details). It follows that if σ is sufficiently large, then

$$sign\left[\frac{(h_i^\sigma)'(w_i^\sigma)}{h_i^\sigma(w_i^\sigma)} - \frac{h_i^\sigma(w_i^\sigma)}{H_i^\sigma(w_i^\sigma)}\right] = sign\left[\frac{(1 - G_i(z_i^*))f_i'(v_i)}{f_i(v_i)} - \frac{f_i(v_i)}{F_i(v_i)}\right].$$

Since $(\log H_i{}^{\sigma})'' = ((h_i^{\sigma})' H_i^{\sigma} - (h_i^{\sigma})^2)/(H_i^{\sigma})^2$, H_i^{σ} is log-concave if $(h_i^{\sigma})'/h_i^{\sigma} - h_i^{\sigma}/H_i^{\sigma} < 0$ for all w_i^{σ}. Given w_i^{σ}, this inequality holds when σ is large because the right-hand side of the displayed equation is negative by log-concavity of F_i. In the online appendix, we prove that there exists $\bar{\sigma}$ such that if $\sigma > \bar{\sigma}$, then the right-hand side is negative for all w_i^{σ}.

(3) If $f_i'(u_0 + c_i - z_i^*) \leq 0$, then H_i is log-concave above $w_i > \max\{u_0 + c_i, \underline{w}_i\}$.

Log-concavity of f_i implies that there exists v_i^* such that $f_i'(v_i) \leq 0$ if and only if $v_i \geq v_i^*$. If $f_i'(u_0 + c_i - z_i^*) \leq 0$, then it must be that $u_0 + c_i - z_i^* \geq v_i^*$ and, therefore, $f_i(v_i)$ decreases in v_i whenever $v_i \geq u_0 + c_i - z_i^* \geq v_i^*$. Differentiating equation (6) with respect to w_i, then

$$h_i(w_i) \equiv H_i'(w_i) = \int_{\underline{z}_i}^{\overline{z}_i} f_i(w_i - \min\{z_i, z_i^*\}) g_i(z_i) dz_i.$$

Suppose $w_i > u_0 + c_i$. If w_i rises, then $f_i(w_i - \min\{z_i, z_i^*\})$ falls for any $z_i \in [\underline{z}_i, \overline{z}_i]$, because

$$w_i - \min\{z_i, z_i^*\} \geq w_i - z_i^* > u_0 + c_i - z_i^*.$$

It follows that $h_i(w_i)$ also decreases in w_i as long as $w_i > u_0 + c_i$. It is then automatic that $h_i(w_i)/H_i(w_i)$ falls in w_i above $w_i > u_0 + c_i$. ■

Proof of Proposition 3. We prove that the right-hand side of equation (4) rises in p^*. Let $D_i(p_i, p^*, u_0)$ denote demand for seller i when all other sellers choose p^* and consumers' outside option is given by u_0. It suffice to show the following cross-derivative is positive:

$$\frac{d}{dp^*}\left(-\frac{\partial log(D_i(p_i, p^*, u_0))}{\partial p_i}|_{p_i=p^*}\right) = \frac{d}{dp^*}\left(-\frac{\partial log(D_i(p_i + u_0, p^* + u_0, 0)}{\partial p_i}|_{p_i=p^*}\right)$$
$$= \frac{d}{du_0}\left(-\frac{\partial log(D_i(p_i + u_0, p^* + u_0, 0)}{\partial p_i}|_{p_i=p^*}\right) = -\frac{\partial^2 log(D_i(p_i, p^*, u_0))}{\partial p_i \partial u_0}|_{p_i=p^*} \geq 0.$$

Note that the first and the third equations use the fact that $D_i(p_i, p^*, u_0) = D_i(p_i + u_0, p^* + u_0, 0)$. The inequality holds because D_i is log-submodular in (p_i, u_0) by Theorem 1 in Quint (2014). ■

Proof of Proposition 4.

(1) If Z becomes more dispersed, then $\min\{Z, z^*\}$ also becomes more dispersed.

Notice that the quantile function for $\min\{Z, z^*\}$ is given by $\min\{G^{-1}(a), z^*\}$. It suffices to show that its slope increases at any $a \in (0, 1)$. For $a < G(z^*)$, the result is immediate from $\min\{G^{-1}(a), z^*\} = G^{-1}(a)$. For $a \geq G(z^*)$, the result follows from the fact that $G(z^*)$ rises as G becomes more dispersed: rewriting equation (1) with $b^* = G(z^*)$ and $b = G(z)$ yields $s = \int_{b^*}^{1}(1-b)\partial G^{-1}(b)/\partial b db$. If G becomes more dispersed ($\partial G^{-1}(b)/\partial b$ rises), the integrand rises, and thus the lower support b^* must rise in order to maintain the equation.[27]

(2) Fix V_0 and consider V_1 that is more dispersed than V_0. For each $i = 0, 1$, let F_i denote the distribution function for V_i. Assume that both f_0 and f_1 are decreasing above $v \geq u_0 + c - z^*$ and the lower bound of their support is given by $\underline{v} \in (-\infty, u_0 + c - z^*]$. Define the quantile $\underline{a}_0 \equiv Pr\{V_0 + \min\{Z, z^*\} \leq u_0 + c - z^*\}$. In the following, we show that the quantile function of $V_1 + \min\{Z, z^*\}$ is steeper than that of $V_0 + \min\{Z, z^*\}$ for all quantiles above $\underline{a}_0$.

For $t \in [0, 1]$, let V_t be the random variable whose quantile function is given by $F^{-1}(a, t) \equiv (1-t)F_0^{-1}(a) + tF_1^{-1}(a)$. It is clear that V_t grows more dispersed in t. Since V_1 dominates V_0 in the sense of first-order stochastic dominance (Theorem 3.B.13 in Shaked and Shanthikumar (2007)), V_t also rises, in the sense of first-order stochastic dominance, as t rises. Let $F(v, t)$ and $f(v, t)$ denote the distribution function and the density function of V_t, respectively. Given t and $\underline{b}_0 \equiv F_0(u_0 + c - z^*)$, since V_1 dominates V_0 in the first-order stochastic dominance sense and f_0 and f_1 have decreasing density, $f_v(v, t) \leq 0$ for any $v \geq F^{-1}(\underline{b}_0, t)$.

Let $W_t \equiv V_t + \min\{Z, z^*\}$, and denote by $H(w, t)$ and $H^{-1}(a, t)$ its distribution function and quantile function, respectively. To prove that $\partial H^{-1}(a, t)/\partial a$ rises in t at all $a \in [\underline{a}_0, 1)$, it suffices to show that $H_t(w, t)/h(w, t)$ falls monotonically in w for $w \geq H^{-1}(\underline{a}_0, t)$, because given $a = H(w)$, $\partial[\partial H^{-1}(a, t)/\partial a]/\partial t = \partial[\partial H^{-1}(a, t)/\partial t]/\partial a = -\partial[H_t(w, t)/h(w, t)]/\partial a$ has the same sign as $-\partial[H_t(w, t)/h(w, t)]/\partial w$. Notice that since $u_0 + c - z^* > \underline{v}$, $w \geq \underline{v} + z^*$ whenever $w \geq u_0 + c$ (i.e., whenever w lies in the relevant region). Given $w \geq \underline{v} + z^*$, by equation (2), $H_t(w, t)/h(w, t)$ can be written as

$$\frac{H_t(w, t)}{h(w, t)} = \frac{\int_{\underline{z}}^{\bar{z}} F_t(w - \min\{z, z^*\}, t)g(z)dz}{\int_{\underline{z}}^{\bar{z}} f(w - \min\{z, z^*\}, t)g(z)dz} \tag{7}$$
$$= \frac{\int_{\underline{z}-w}^{\bar{z}-w} \frac{F_t(\max\{-r, w-z^*\}, t)}{f(\max\{-r, w-z^*\}, t)} f(\max\{-r, w - z^*\}, t)g(r + w)dr}{\int_{\underline{z}-w}^{\bar{z}-w} f(\max\{-r, w - z^*\}, t)g(r + w)dr} = E\left[\frac{F_t(\max\{-R, w - z^*\}, t)}{f(\max\{-R, w - z^*\}, t)}\right].$$

The second equation is due to a change of variables with $r = z - w$. The random variable R in

[27] This argument is borrowed from Choi and Smith (2016).

the last equation has density $f(\max\{-r, w-z^*\}, t)g(r+w)$ and support $(\underline{z}-w, \bar{z}-w)$. The right-hand side falls in w for two reasons. First, as explained above with H, $F_t(v,t)/f(v,t)$ falling in v is equivalent to $F(v,t)$ growing more dispersed in t. So the ratio inside the expectation operator falls in w for any given R. Second, R falls in the first-order stochastic dominance sense in w because (i) the support of R shifts to the left in w and (ii) the density of R is log-submodular in (r,w) by log-concavity of g and by $f_v(w-z^*,t) \leq 0$ for $w \geq H^{-1}(\underline{a}_0, t)$. To see the inequality, recall that $f_v(w-z^*,t) \leq 0$ for $w - z^* \geq F^{-1}(\underline{b}_0, t)$. One can show $H^{-1}(\underline{a}_0, t) \geq F^{-1}(\underline{b}_0, t) + z^*$ for each $t \in [0,1]$: For $t = 0$, $H^{-1}(\underline{a}_0, 0) = u_0 + c$ and $F^{-1}(\underline{b}_0, 0) = u_0 + c - z^*$ by the definition of $\underline{a}_0$ and $\underline{b}_0$, and thus the inequality binds. For $t > 0$, the inequality holds as $dH^{-1}(\underline{a}_0, t)/dt \geq dF^{-1}(\underline{b}_0, t)/dt$ whenever the inequality binds.[28] ■

Proof of the profit result in Proposition 5. Let $\mathbf{p}^* = (p_i^*, p_{-i}^*)$ denote the equilibrium price vector. In addition, in a slight abuse of notation, let $D_i(\mathbf{p}^*, z^*)$ denote the equilibrium demand for seller i given $\mathbf{p}^*$ and z^*, and $\pi_i(\mathbf{p}^*, z^*)$ the corresponding profit (i.e., $\pi_i(\mathbf{p}^*, z^*) = (p_i^* - c)D_i(\mathbf{p}^*, z^*)$). By Theorem 1, $D_i(\mathbf{p}^*, z^*) = Pr\{V_i + \min\{Z_i, z^*\} - p_i^* > X_i\}$ where $X_i \equiv \max\{u_0, \max_{j\neq i} V_j + \min\{Z_j, z^*\} - p_j^*\}$. Let $d\pi_i(\mathbf{p}^*, z^*)/ds$ represent the effect of a marginal increase in s on each seller's equilibrium profit. Then,

$$\frac{d\pi_i(\mathbf{p}^*, z^*)}{ds} = \frac{\partial p_i^*}{\partial s}\frac{\partial \pi_i(\mathbf{p}, z^*)}{\partial p_i}|_{\mathbf{p}=\mathbf{p}^*} + \frac{\partial p_{-i}^*}{\partial s}\frac{\partial \pi_i(\mathbf{p}, z^*)}{\partial p_{-i}}|_{\mathbf{p}=\mathbf{p}^*} + \frac{\partial z^*}{\partial s}\frac{\partial \pi_i(\mathbf{p}, z^*)}{\partial z^*}|_{\mathbf{p}=\mathbf{p}^*}.$$

Each term represents the marginal effect of own price, that of the other sellers' prices, and that of consumer search behavior, respectively. In equilibrium, the first term is 0 by the envelope theorem ($\partial \pi_i(\mathbf{p}, z^*)/\partial p_i = 0$ at $\mathbf{p} = \mathbf{p}^*$). The second term is negative because $\partial p^*/\partial s \leq 0$, as shown above, and $\partial \pi_i(\mathbf{p}, z^*)/\partial p_{-i} \geq 0$, as the products are imperfect substitutes one another. The last term is negative because $\partial z^*/\partial s < 0$ (equation (1)) and $\partial \pi_i(\mathbf{p}, z^*)/\partial z^* \geq 0$: the latter inequality is due to the fact that an increase in z^* raises H in the sense of first-order stochastic dominance, induces fewer consumers to take the outside option and, therefore, raises $D_i(\mathbf{p}^*, z^*)$. Overall, it is necessarily the case that $d\pi(\mathbf{p}^*, z^*)/ds \leq 0$. ■

Proof of Proposition 6. Given that there is no outside option, the equilibrium price p^* is

[28] By the implicit function theorem, $dH^{-1}(\underline{a}_0, t)/dt = -H_t(w,t))/h(w,t)$ where $w = H^{-1}(\underline{a}_0, t)$, and similarly $dF^{-1}(\underline{b}_0, t)/dt = -F_t(w,t))/f(w,t)$ where $w = F^{-1}(\underline{b}_0, t)$. If $H^{-1}(\underline{a}_0, t) = F^{-1}(\underline{b}_0, t) + z^*$, then $dH^{-1}(\underline{a}_0, t)/dt \geq dF^{-1}(\underline{b}_0, t)/dt$ by equation (7) and the fact that $F_t(w,t)/f(w,t)$ falls in w.

given by

$$\frac{1}{p^* - c} = n \int_{\underline{w}}^{\infty} h(w) dH(w)^{n-1} = n \int_0^1 h(H^{-1}(a)) da^{n-1},$$

where the second equation changes variable $a = H(w)$. By the implicit function theorem,

$$\frac{\partial p^*}{\partial \alpha} = -(p^* - c)^2 n \int_0^1 \frac{\partial h(H^{-1}(a))}{\partial \alpha} da^{n-1}.$$

The desired result follows by letting $\gamma(a) = \partial h(H^{-1}(a))/\partial \alpha$ and applying the next two results. Lemma 1 shows that $\gamma(a)$ is positive and then negative as a rises. Lemma 2 uses Lemma 1 to show that $\int_0^1 \gamma(a) da^n < 0$ if and only if n is large. ■

Lemma 1 *There exists $w^*(< \infty)$ such that the slope of $H^{-1}(a)$ decreases in α if and only if $a > H(w^*)$.*

The complete proof is long and is in the online appendix (Section G). Intuitively, if w is large, then the shape of H is mostly determined by F. Since F becomes more dispersed in α, H also does so for w large. If w is small, then H is affected by all three V, Z, and z^*. The effects of the first two cancel each other out, because $V + Z \sim N(0, 1)$. The last effect through z^*, however, makes H less dispersed, because z^* decreases in α (see equation (1) and the proof of Proposition 5).

Lemma 2 *For any real-valued function $\gamma : \mathcal{R} \to \mathcal{R}$, if $\int_0^1 \gamma(a) da^n \leq 0$ and there exists a' such that $\gamma(a) < 0$ if and only if $a > a'$, then*

$$\int_0^1 \gamma(a) da^{n+1} = \frac{n+1}{n} \int_0^1 \gamma(a) a da^n \leq 0.$$

Proof. The result immediately follows from the fact that a is positive and strictly increasing and, therefore, assigns more weight to the negative portion of $\gamma(a)$ in the integral (see Karlin and Rubin (1955) for a formal proof of this logic). ■

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Supplement to "Consumer Search and Price Competition"

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A Distributions of Effective Values

In this appendix, we provide three examples in which $H_i(w_i)$ can be explicitly calculated.

(1) Uniform: suppose V_i and Z_i are uniform over $[0,1]$ (i.e., $F_i(v) = G_i(v) = v$). Provided that $s \leq 1/2$ (which guarantees $z_i^* \in [0,1]$), $z_i^* = 1 - \sqrt{2s}$. It is then straightforward to show that $H_i(w_i)$ is given as follows:

$$H_i(w_i) = \begin{cases} \frac{w_i^2}{2}, & \text{if } w_i \in [0, z_i^*), \\ w_i - z_i^* + \frac{(z_i^*)^2}{2}, & \text{if } w_i \in [z_i^*, 1), \\ 2w_i - \frac{w_i^2}{2} - z_i^* + \frac{(z_i^*)^2}{2} - \frac{1}{2}, & \text{if } w_i \in [1, 1+z_i^*]. \end{cases}$$

Notice that, whereas H_i is continuous, the density function h_i has an upward jump at z_i^*. Therefore, H_i is not globally log-concave. Nevertheless, it is easy to show that both H_i and $1 - H_i$ are log-concave above z_i^*.

(2) Exponential: suppose V_i and Z_i are exponential distributions with parameters λ_1 and λ_2, respectively (i.e., $F_i(v_i) = 1 - e^{-\lambda_1 v_i}$ and $G_i(z_i) = 1 - e^{-\lambda_2 z_i}$). Provided that $s < 1/\lambda_2$ (which ensures that $z_i^* > 0$), then $z_i^* = -\log(\lambda_2 s)/\lambda_2$. For any $w_i \geq 0$,

$$H_i(w_i) = 1 - e^{-\lambda_2 \min\{w_i, z_i^*\}} - \frac{\lambda_2 (e^{(\lambda_1 - \lambda_2)\min\{w_i, z_i^*\}} - 1)}{e^{\lambda_1 w_i}(\lambda_1 - \lambda_2)} + (1 - e^{-\lambda_1(\max\{w_i, z_i^*\} - z_i^*)}) e^{-\lambda_2 z_i^*}.$$

Similar to the uniform example, H_i is not globally log-concave, because h_i has a upward jump at z_i^*, but both H_i and $1 - H_i$ are log-concave above z_i^*.

(3) Gumbel: suppose that V_i and $-Z_i$ are standard Gumbel distributions (i.e., $F_i(v_i) =$

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$e^{-e^{-v_i}}$ and $G_i(z_i) = 1 - e^{-e^{z_i}}$). For any $w_i \in (-\infty, \infty)$,

$$H_i(w_i) = \frac{1 + e^{-w_i - e^{z_i^*}(1+e^{-w_i})}}{1 + e^{-w_i}}.$$

Since both f_i and g_i are log-concave, $1 - H_i$ is log-concave by Proposition 2. Given the solution for H_i above, we have

$$\frac{h_i(w_i)}{H_i(w_i)} = \frac{e^{z_i^* - w_i} - 1}{1 + e^{w_i + e^{z_i^*}(1+e^{-w_i})}} + \frac{1}{1 + e^{w_i}}.$$

The first term falls in w_i whenever $w_i \geq z_i^*$, while the second term constantly falls in w_i. Therefore, $H_i(w_i)$ is log-concave above z_i^*.

B Proof of the second claim in Proposition 2 (Cont'd)

Since

$$(logH_i^\sigma(w_i^\sigma))'' = \frac{(h_i^\sigma)'(w_i^\sigma)H_i^\sigma(w_i^\sigma) - h_i^\sigma(w_i^\sigma)^2}{H_i^\sigma(w_i^\sigma)^2},$$

it suffices to show that $(h_i^\sigma)'(w_i^\sigma)H_i^\sigma(w_i^\sigma) - h_i^\sigma(w_i^\sigma)^2 < 0$ for all w_i^σ, provided that σ is sufficiently large. Integrate equation (2) by parts, we have $H_i^\sigma(w_i^\sigma) = \int_{\underline{v}_i^\sigma}^{\bar{v}_i^\sigma} G_i(w_i^\sigma - v_i^\sigma)dF_i^\sigma(v_i^\sigma)$ for $w_i^\sigma < \underline{v}_i^\sigma + z_i^*$. In this case H_i^σ is log-concave by Prékopa's Theorem. For $w_i^\sigma \geq \underline{v}_i^\sigma + z_i^*$, we have

$$H_i^\sigma(w_i^\sigma) = \int_{w_i^\sigma - z_i^*}^{\bar{v}_i^\sigma} G_i(w_i^\sigma - v_i^\sigma)dF_i^\sigma(v_i^\sigma) + F_i^\sigma(w_i^\sigma - z_i^*).$$

By straightforward calculus,

$$\frac{h_i^\sigma(w_i^\sigma)}{H_i^\sigma(w_i^\sigma)} = \frac{\int_{w_i^\sigma - z_i^*}^{\bar{v}_i^\sigma} g_i(w_i^\sigma - v_i^\sigma)dF_i^\sigma(v_i^\sigma) + (1 - G_i(z_i^*))f_i^\sigma(w_i^\sigma - z_i^*)}{\int_{w_i^\sigma - z_i^*}^{\bar{v}_i^\sigma} G_i(w_i^\sigma - v_i^\sigma)dF_i^\sigma(v_i^\sigma) + F_i^\sigma(w_i^\sigma - z_i^*)}.$$

Changing the variables with $a = F_i^\sigma(v_i^\sigma)$ and $r = F_i^\sigma(w_i^\sigma - z_i^*)$, the above equation becomes

$$\frac{h_i^\sigma((F_i^\sigma)^{-1}(r) + z_i^*)}{H_i^\sigma((F_i^\sigma)^{-1}(r) + z_i^*)} = \frac{\int_r^1 g_i((F_i^\sigma)^{-1}(r) - (F_i^\sigma)^{-1}(a) + z_i^*)da + (1 - G_i(z_i^*))f_i^\sigma((F_i^\sigma)^{-1}(r))}{\int_r^1 G_i((F_i^\sigma)^{-1}(r) - (F_i^\sigma)^{-1}(a) + z_i^*)da + r}.$$

Since $V_i^\sigma \equiv \sigma V_i$, we have $F_i^\sigma(v_i^\sigma) = F_i(v_i^\sigma/\sigma)$, $(F_i^\sigma)^{-1}(r) = \sigma F_i^{-1}(r)$, $f_i^\sigma((F_i^\sigma)^{-1}(r)) = f_i(F_i^{-1}(r))/\sigma$, and $(f_i^\sigma)'(F_i^{-1}(r)) = f_i(F_i^{-1}(r))/\sigma^2$. Arranging the terms in the right-hand side above yields

$$\frac{\sigma h_i^\sigma((F_i^\sigma)^{-1}(r)+z_i^*)}{H_i^\sigma((F_i^\sigma)^{-1}(r)+z_i^*)} = \frac{\int_r^1 \sigma g_i(\sigma(F_i^{-1}(r)-F_i^{-1}(a))+z_i^*)da + (1-G_i(z_i^*))f_i(F_i^{-1}(r))}{\int_r^1 G_i(\sigma(F_i^{-1}(r)-F_i^{-1}(a))+z_i^*)da + r}.$$

Since $F_i^{-1}(r) - F_i^{-1}(a) \le 0$, the denominator converges to r as σ explodes. Integrating $\int_r^1 \sigma g_i(\sigma(F_i^{-1}(r)-F_i^{-1}(a))+z_i^*)da$ in the numerator by parts yields

$$G_i(z_i^*)f_i(F^{-1}(r)) + \int_r^1 G_i(\sigma(F_i^{-1}(r)-F_i^{-1}(a))+z_i^*)df(F_i^{-1}(a)).$$

Again, since $F_i^{-1}(r) - F_i^{-1}(a) \le 0$, the second term vanishes as σ tends to infinity, and thus the numerator converges to $G_i(z_i^*)f_i(F_i^{-1}(r))$. Therefore,

$$\lim_{\sigma\to\infty} \frac{\sigma h_i^\sigma((F_i^\sigma)^{-1}(r)+z_i^*)}{H_i^\sigma((F_i^\sigma)^{-1}(r)+z_i^*)} = \frac{f_i(F_i^{-1}(r))}{r}.$$

Following a similar procedure, we have

$$\lim_{\sigma\to\infty} \frac{\sigma (h_i^\sigma)'((F_i^\sigma)^{-1}(r)+z_i^*)}{h_i^\sigma((F_i^\sigma)^{-1}(r)+z_i^*)} = \frac{(1-G_i(z_i^*))f_i'(F_i^{-1}(r))}{f_i(F_i^{-1}(r))}.$$

Altogether,

$$\begin{aligned} & \lim_{\sigma\to\infty} \sigma \left[\frac{(h_i^\sigma)'((F_i^\sigma)^{-1}(r)+z_i^*)}{h_i^\sigma((F_i^\sigma)^{-1}(r)+z_i^*)} - \frac{h_i^\sigma((F_i^\sigma)^{-1}(r)+z_i^*)}{H_i^\sigma((F_i^\sigma)^{-1}(r)+z_i^*)} \right] \\ = \; & \frac{(1-G_i(z_i^*))f_i'(F_i^{-1}(r))}{f_i(F_i^{-1}(r))} - \frac{f_i(F_i^{-1}(r))}{r} \\ = \; & (1-G_i(z_i^*)) \left[\frac{f_i'(F_i^{-1}(r))}{f_i(F_i^{-1}(r))} - \frac{f_i(F_i^{-1}(r))}{r} \right] - \frac{G_i(z_i^*)f_i(F_i^{-1}(r))}{r} < 0. \end{aligned} \tag{8}$$

Provided s_i is not too large, then $G_i(z_i^*)$ and $1-G_i(z_i^*)$ are in $(0,1)$, so the sign of the expression is determined by both terms.[29] The square bracket term is weakly negative because F is log-concave, thus the entire expression is weakly negative. The strict inequality (8) holds for

[29] If s_i is large so that $G_i(z_i^*) = 0$, then $W_i = V_i + z_i^*$ and H_i has the same shape as F_i, and thus is log-concave.

each $r \in [0,1]$ because $f_i(F_i^{-1}(r))/r > 0$ when $r \in [0,1)$ and $f_i'(F_i^{-1}(r))/f_i(F_i^{-1}(r)) < 0$ when $r = 1$.[30] Altogether, for each $r \in [0,1]$ there is a $\bar{\sigma}_r < \infty$ such that if $\sigma > \bar{\sigma}_r$, then $(logH_i^\sigma(w_i^\sigma))'' \propto (h_i^\sigma)'(w_i^\sigma)/h_i^\sigma(w_i^\sigma) - h_i^\sigma(w_i^\sigma)/H_i^\sigma(w_i^\sigma) < 0$ where $w_i^\sigma = F_i^{-1}(r) + z_i^*$. Since $[0,1]$ is a compact convex set and $(logH_i^\sigma(w_i^\sigma))''$ is continuous in r, there exists $\bar{\sigma} = \max_{r\in[0,1]} \bar{\sigma}_r < \infty$ such that if $\sigma > \bar{\sigma}$, then $(h_i^\sigma)'/h_i^\sigma - h_i^\sigma/H_i^\sigma < 0$ for all $r \in [0,1]$, or equivalently $H_i^\sigma(w)$ is log-concave for all $w_i^\sigma \geq \underline{v}_i^\sigma + z_i^*$. Finally, if $f_i(\underline{v}_i) = 0$, then the ratio $h_i^\sigma(w_i^\sigma)/H_i^\sigma(w_i^\sigma)$ is continuous at $\underline{v}_i^\sigma + z_i^*$. Since this ratio is decreasing for $w_i < \underline{v}_i^\sigma + z_i^*$ and decreasing for $w_i \geq \underline{v}_i^\sigma + z_i^*$ when σ is large, it is globally decreasing when σ is large, or equivalently $H_i^\sigma(w_i^\sigma)$ is globally log-concave. ■

C Example of a Mixed-strategy Equilibrium

Now we assume F_i is degenerate and characterize a symmetric mixed-strategy equilibrium. Assume there are two symmetric sellers and $u_0 = c_i = v_i = 0$. Assume Z_i is exponentially distributed with parameter λ, namely $G_i(z) = 1 - e^{-\lambda z}$. Assume $s < 1/\lambda$ so that $z^* > 0$. Below we characterize the distribution of prices and show that it has decreasing density.

Let $Q_i = \min\{Z_i, z^*\} - P_i$, and let Γ_i and γ_i be its distribution function and density function, repectively. Note that the equilibrium price P_i is ex-ante random in a mixed-strategy equilibrium. Moreover, in a symmetric equilibrium, the distribution of P_i has no mass point, for if it has a mass point then a seller can get an upward jump in demand by moving the location of the mass point slightly to the left. Since the density of P_i exists (its cdf is atomless), the density γ_i also exists.

First we derive the demand function in a mixed-strategy equilibrium. By the eventual purchase theorem consumers buy from seller 1 if $\min\{z^*, Z_1\} - p_1 > \max\{Q_2, 0\}$. Therefore, no consumer will buy from seller 1 if $p_1 > z^*$. For all $p_1 \leq z^*$, consumers buy from seller 1 when $z^* - p_1 > Q_2$ and $Z_1 - p_1 > \max\{Q_2, 0\}$. Therefore, for all $p_1 \leq z^*$, seller

[30] For $r \in (0,1)$, the strict inequality (8) is true as $f_i(F_i^{-1}(r)) > 0$ within the support. Since $f_i(F_i^{-1}(r))/r$ falls in r by log-concavity of F_i, $f_i(F_i^{-1}(r))/r > 0$ at $r = 0$, and thus the strict inequality (8) also holds for $r = 0$. For $r = 1$, since f_i has unbounded upper support, $f_i(F_i^{-1}(r))$ falls in r when r is large. Therefore $f_i'(F_i^{-1}(r))/f_i(F_i^{-1}(r)) < 0$ for some $r \in (0,1)$. Since $f_i'(F_i^{-1}(r))/f_i(F_i^{-1}(r))$ falls in r by the log-concavity of f_i, $f_i'(F_i^{-1}(r))/f_i(F_i^{-1}(r)) < 0$ when $r = 1$ and thus the inequality (8) holds when $r = 1$.

1's demand and its derivative are given by

$$D_1(p_1) = \int_{\underline{q}}^{z^*-p_1} (1 - G(p_1 + \max\{q, 0\}))d\Gamma_2(q) = \int_{\underline{q}}^{z^*-p_1} e^{-\lambda(p_1+\max\{q,0\})} d\Gamma_2(q),$$

$$D_1'(p_1) = -e^{-\lambda z^*}\gamma_2(z^* - p_1) - \lambda \int_{\underline{q}}^{z^*-p_1} e^{-\lambda(p_1+\max\{q,0\})} d\Gamma_2(q).$$

Therefore, the first-order necessary condition with respect to p_1 is

$$\frac{1}{p_1} = \frac{-D_1'(p_1)}{D_1(p_1)} = \frac{e^{-\lambda z^*}\gamma_2(z^* - p_1)}{D_1(p_1)} + \lambda.$$

Let π^* be the equilibrium profit for the sellers in a symmetric equilibrium. Since seller 1 is indifferent between offering any prices in the support of P_1 in equilibrium, $\pi^* = p_1 D(p_1)$ for every p_1 in the support of P_1. Using $D_1(p_1) = \pi^*/p_1$, the first-order condition can be rewritten as

$$\gamma_2(z^* - p_1) = \frac{\pi^*}{p_1}\left(\frac{1}{p_1} - \lambda\right) e^{\lambda z^*}. \tag{9}$$

The first-order condition implies $p_1 \leq 1/\lambda$ in equilibrium. Since $p_1 \geq 0$, the support of P_1 is a subset of the interval $[0, \min\{z^*, 1/\lambda\}]$. From equation (9) it is clear that the density γ_i of Q_i is monotonically increasing (because the right-hand side falls in p_1).

Now we use the density of Q_i (i.e. γ_i) and that of Z_i to solve for the distribution of P_i, by exploiting the equation $Q_i = \min\{Z_i, z^*\} - P_i$. This is generally a hard problem because one must solve a complex differential equation. Below we show that the problem is especially tractable when Z_i is exponentially distributed. Let $B(p)$ be the distribution function of P_i in a symmetric equilibrium. The cdf and pdf of Q_i can be written as

$$\Gamma_i(q) = \int_0^\infty [1 - B(\min\{z, z^*\} - q)]\lambda e^{-\lambda z} dz,$$

$$\gamma_i(q) \equiv \Gamma_i'(q) = \int_0^\infty b(\min\{z, z^*\} - q)\lambda e^{-\lambda z} dz.$$

Substitute the equation for γ_i into the first-order condition (9), then

$$\frac{\pi^*}{p}\left(\frac{1}{p} - \lambda\right) e^{\lambda z^*} = \int_0^\infty b(\min\{z - z^* + p, p\})\lambda e^{-\lambda z} dz = \int_{-z^*}^0 b(y + p)\lambda e^{-\lambda(y+z^*)} dy + b(p)e^{-\lambda z^*}.$$

The last line uses a change of variable $y = z - z^*$. Now multiply both sides by $e^{\lambda(z^* - p)}$, and let $\tau(p) \equiv b(p)e^{-\lambda p}$ and $T(p) \equiv \int_0^p \tau(y)dy$. Then we can rewrite the above equation as

$$\frac{\pi^*}{p}\left(\frac{1}{p} - \lambda\right)e^{\lambda(2z^* - p)} = \lambda \int_{-z^*}^{0} \tau(y + p)dy + \tau(p).$$

Notice that, since $p \geq 0$ in equilibrium, the density $b(q) = \tau(q) = 0$ for all $q < 0$. Together with $p \leq z^*$, we have $\tau(y + p) = 0$ for all $y \in (-z^*, -p)$. In light of this, the lower support of the integral term can be replaced by $-p$. Therefore, the equation above becomes

$$\frac{\pi^*}{p}\left(\frac{1}{p} - \lambda\right)e^{\lambda(2z^* - p)} = \lambda \int_{-p}^{0} \tau(y + p)dy + \tau(p) = \lambda T(p) + \tau(p). \tag{10}$$

This equation is a first-order differential equation. The general solution is

$$T(p) = Ce^{-\lambda p} - \pi^* e^{\lambda(2z^* - p)}\left(\lambda \log(p) + \frac{1}{p}\right)$$

where C is a constant. By $b(p) = \tau(p)e^{\lambda p}$ and equation (10), the density $b(p)$ is

$$b(p) = \frac{\pi^*}{p}\left(\frac{1}{p} - \lambda\right)e^{2\lambda z^*} - \lambda T(p)e^{\lambda p} = \pi^* e^{2\lambda z^*}\left(\frac{1}{p^2} + \lambda^2 \log(p)\right) - \lambda C.$$

The constant C is chosen so that $\int_0^{\min\{z^*, 1/\lambda\}} b(p)dp = 1$. The value of π^* can be solved by substituting the solution of $b(p)$ into the seller's profit function. One can easily show that the density $b(p)$ falls in p by the equation above and $p \leq 1/\lambda$.

D Unobservable Prices and Search Costs

Anderson and Renault (1999) study a stationary search model with unobservable prices, and show that $\partial p^*/\partial s > 0$ provided that $1 - G(z)$ is log-concave. We argue that this insight may not hold when search is non-stationary, due to the presence of a prior value V. Assume there is no outside option and sellers are symmetric. Below we show $\partial p^*/\partial s < 0$ is possible if the density of V is log-concave and increasing, even when $1 - G(z)$ is log-concave.

Claim 1 *The equilibrium price p^* falls in s when (i) s is sufficiently small and (ii) $f'(\bar{v})/f(\bar{v}) > \lim_{z \uparrow \bar{z}} g(z)/[1-G(z)]$.*

Since we have assumed $f(v)$ is log-concave, it is single-peaked in v. Therefore, the second condition requires $f'(v) > 0$ for all $[\underline{v}, \bar{v}]$, and the upper support $\bar{v}$ must be finite.

Proof. Let $\tilde{W}_i \equiv \max_{j \neq i} W_j$, then the demand for seller i is given by (5). When prices are unobservable, seller i controls p_i but not p_i^e, so the measure of marginal consumers is

$$\begin{aligned} -\frac{dD_i(p_i, p_i^e, p^*)}{dp_i}|_{p_i=p_i^e=p^*} &= E\left[\int_{\tilde{W}_i - z^*}^{\bar{v}} g(\tilde{W} - v_i) dF(v_i)\right] \\ &= \int_{\underline{w}}^{\bar{v}+z^*} \left[\int_{w-z^*}^{\bar{v}} g(w - v_i) dF(v_i)\right] dH(w)^{n-1}. \end{aligned}$$

In a symmetric equilibrium, p^* solves

$$p^* - c = -\left(n \frac{dD_i(p_i, p_i^e, p^*)}{dp_i}|_{p_i=p_i^e=p^*}\right)^{-1}.$$

Since the right-hand side does not depend on p^*, to show $\partial p^*/\partial s < 0$, it suffices to show the right-hand side falls in s, or equivalently the following derivative is positive.

$$\begin{aligned} &\frac{d}{ds} \int_{\underline{w}}^{\bar{v}+z^*} \left[\int_{w-z^*}^{\bar{v}} g(w - v_i) dF(v_i)\right] dH(w)^{n-1} \\ =&\frac{dz^*}{ds} \int_{\underline{w}}^{\bar{v}+z^*} \left[g(z^*) f(w - z^*)\right] dH(w)^{n-1} \\ &+ \int_{\underline{w}}^{\bar{v}+z^*} \left[\int_{w-z^*}^{\bar{v}} g(w - v_i) dF(v_i)\right] \left[\frac{f'(w - z^*)}{h(w)} + \frac{(n-2) f(w - z^*)}{H(w)}\right] dH(w)^{n-1}. \end{aligned}$$

The last line uses $dH(w)/ds = f(w - z^*)$ and $dh(w)/ds = f'(w - z^*)$. Next, substitute $dz^*/ds = -1/[1 - G(z^*)]$ (by equation (1)) into the derivative and divide the entire expression by $\int_{\underline{w}}^{\bar{v}+z^*} f(w-z^*)dH(w)^{n-1}$, then the expression above has the same sign as

$$\frac{-g(z^*)}{1-G(z^*)}+\frac{\int_{\underline{w}}^{\bar{v}+z^*}\left[\int_{w-z^*}^{\bar{v}} g(w-v_i)dF(v_i)\right]\left[\frac{f'(w-z^*)}{h(w)}+\frac{(n-2)f(w-z^*)}{H(w)}\right]dH(w)^{n-1}}{\int_{\underline{w}}^{\bar{v}+z^*} f(w-z^*)dH(w)^{n-1}}$$

$$\geq\frac{-g(z^*)}{1-G(z^*)}+\frac{\int_{\underline{w}}^{\bar{v}+z^*}\left[\frac{\int_{w-z^*}^{\bar{v}} g(w-v_i)dF(v_i)}{h(w)}\right]\left[\frac{f'(w-z^*)}{f(w-z^*)}\right]f(w-z^*)dH(w)^{n-1}}{\int_{\underline{w}}^{\bar{v}+z^*} f(w-z^*)dH(w)^{n-1}}.$$

Now take $s\to 0$ and therefore $z^*\to\bar{z}$. Since (i) $h(w)\to\int_{w-z^*}^{\bar{v}} g(w-v_i)dF(v_i)$ as $z^*\to\bar{z}$,[31] and (ii) $f'(\bar{v})/f(\bar{v})\leq f'(v)/f(v)$ for all $v<\bar{v}$ by the log-concavity of f, the limit of the above expression is at least

$$\lim_{z^*\uparrow\bar{z}}\frac{-g(z^*)}{1-G(z^*)}+\frac{f'(\bar{v})}{f(\bar{v})}.$$

Finally, if $f'(\bar{v})/f(\bar{v})>\lim_{z^*\uparrow\bar{z}} g(z^*)/[1-G(z^*)]$, then the last line is clearly positive and thus $\partial p^*/\partial s<0$ when s is small.[32] ■

To put this result in context, note that Haan et al. (2017) show that in a symmetric duopoly model with unobservable prices, if F has full support and $1-G$ is log-concave, then $\partial p^*/\partial s>0$. Since Claim 1 allows $n=2$ and log-concave $1-G$, the sign of $\partial p^*/\partial s$ is reversed in Claim 1 precisely because F has a bounded upper support and rising density. Indeed, when $\bar{v}<\infty$ and $f'>0$, as s rises, the upper support of $H(w)$, namely $\bar{v}+z^*$, falls while the density $h(w)$ rises at all $w<\bar{v}+z^*$. As a result, the measure of marginal consumers rises as the other sellers' search costs rise. By this logic, as the other sellers' search costs rise, seller i is willing to lower p_i to attract more marginal consumers. On the other hand, as s_i rises, seller i has an incentive to raise p_i to extract more surplus from the visiting consumers. The overall effect depends on the relative strength of the two effects. We focus on small s because the first effect is relatively stronger when s is small — indeed, the magnitude of the change in the upper support $\partial(\bar{v}+z^*)/\partial s=-1/(1-G(z^*)$ is the largest when $s\approx 0$. When $s\approx 0$, the relative strength of these two effects depend on the ratio f'/f and the hazard rate $g/(1-G)$ respectively. Finally, since $f'(v)/f(v)$ falls in v

[31]Integrate equation (2) by parts and differentiate with respect to w, then $h(w)=\int_{w-z^*}^{\bar{v}} g(w-v_i)dF(v_i)+(1-G(z^*))f(w-z^*)$. The second term vanishes as $z^*\to\bar{z}$.

[32]If $\bar{z}=\infty$, then $\int_{\underline{w}}^{\bar{v}+z^*} f(w-z^*)dH(w)^{n-1}$ vanishes as $s\to 0$, and thus $\lim_{s\to 0}\partial p^*/\partial s=0$. But by continuity the inequality $\partial p^*/\partial s<0$ remains valid for small but strictly positive s.

and $g(z)/(1-G(z))$ rises in z, our second sufficient condition ensures $f'/f > g/(1-G)$ at all v and z.

E Consumer Surplus and Search Costs

We present an example where consumer surplus rises with search costs. Consider a symmetric duopoly environment with no outside option. Assume the prior and match values are uniform random variables with $V \sim U[0, 3/4]$ and $Z \sim U[0, 1]$. Since there is no outside option and $p_1 = p_2 = p^*$ in a symmetric equilibrium, consumers always purchase the product with a higher effective value. In Section F we show that one can directly use the effective values to compute consumer surplus. Using that insight, a (representative) consumer's expected surplus is

$$CS = E[\max\{W_1, W_2\}] - p^*.$$

First consider the effects of s on p^*. The equilibrium price is $p^* = 6/(9 + 32s)$ by direct calculation.[33] This implies

$$\frac{dp^*}{ds} = \frac{-192}{(9 + 32s)^2}.$$

The expected value of the first-order statistic $\max\{W_1, W_2\}$ can be written as

$$E[\max\{W_1, W_2\}] = 2\int_0^1 \int_0^{\frac{3}{4}} (v + \min\{z, z^*\})H(v + \min\{z, z^*\})dvdz.$$

Next, we consider the effect of s on $E[\max\{W_1, W_2\}]$. By equation (1) $dz^*/ds = -1/(1 - z^*)$. This result and the equation above imply

$$\begin{aligned} \frac{dE[\max\{W_1, W_2\}]}{ds} =& -2\int_0^{\frac{3}{4}} [H(v + z^*) + (v + z^*)h(v + z^*)]dv \\ & - \frac{2}{1 - z^*}\int_0^1 \left[\int_0^{\frac{3}{4}} (v + \min\{z, z^*\})H_{z^*}(v + \min\{z, z^*\})dv\right] dz, \end{aligned} \tag{11}$$

[33] This pricing formula is also provided by Haan et al. (2017). They show that $p^* = 3\bar{z}^2\bar{v}/(3\bar{z}\bar{v} + 3s\bar{v} - \bar{v}^2)$, assuming the return to search is sufficiently high so that the consumers who visit seller 1 first will always visit seller 2 with a strictly positive probability. They show that this assumption is satisfied when s is sufficiently small and $\bar{z} > \bar{v}$. Both conditions are satisfied in our example.

where $H_{z^*}(w)$ is defined as

$$H_{z^*}(w) \equiv \frac{dH(w)}{dz^*} = -f(w - z^*)(1 - G(z^*)) = -\frac{4}{3}(1 - z^*) \quad \text{for} \quad w \in [z^*, z^* + 4/3],$$

and otherwise 0.

Now we evaluate the effect of an increase in s on CS at $s = 0$. When $s = 0$, $z^* = 1$ by equation (1). By direct calculation, the density and distribution function of W are

$$h(w) = \begin{cases} 4w/3 & \text{if } w \leq 3/4 \\ 1 & \text{if } 3/4 < w < 1 \\ 7/3 - 4w/3 & \text{if } 7/4 \geq w > 1. \end{cases}$$

$$H(w) = \begin{cases} 2w^2/3 & \text{if } w \leq 3/4 \\ w - 3/8 & \text{if } 3/4 < w < 1 \\ 7w/3 - 2w^2/3 - 25/24 & \text{if } 7/4 \geq w > 1. \end{cases}$$

Substitute the expressions for h, H and H_{z^*} into equation (11), then

$$\begin{aligned} \frac{dE[\max\{W_1, W_2\}]}{ds}|_{s=0} = & -2\left[\int_0^{\frac{3}{4}} [H(v+1) + (v+1)h(v+1)]dv\right] \\ & + \frac{8}{3}\int_0^1 \int_0^{\frac{4}{3}} (v+z)\mathbb{1}_{\{v+z>1\}} dv dz \\ = & -2\left[\int_1^{\frac{7}{4}} -2w^2 + \frac{14}{3}w - \frac{25}{24} dw\right] + \frac{8}{3}\left(\frac{45}{128}\right) = -\frac{21}{16}. \end{aligned}$$

Altogether, a consumer's expected surplus rises in s when $s = 0$ because

$$\frac{dCS}{ds}|_{s=0} = \frac{dE[\max\{W_1, W_2\}]}{ds}|_{s=0} - \frac{dp^*}{ds}|_{s=0} = -\frac{21}{16} + \frac{192}{81} = \frac{457}{432} > 0.$$

Intuitively, as s rises, each consumer pays a larger utility cost to visit sellers. On the other hand they are better off because the equilibrium price p^* falls in s. This example shows that the latter effect can dominate the former when s is small.

F Calculation of Consumer Surplus

In this section we argue that our discrete-choice reformulation is useful not only for predicting shopping outcomes, but can also be used to compute consumer surplus. Let $EU(u_0)$ be a (representative) consumer's expected payoff and $CS(u_0)$ be her surplus from search, namely $CS(u_0) \equiv EU(u_0) - u_0$.

Claim 2 *A consumer's surplus is*

$$CS(u_0) = \int_{u_0}^{\infty} [1 - \Pi_{i=1}^{n} H_i(w + p_i)]dw.$$

This result implies that a representative consumer's surplus is the same as that in a discrete-choice model. Indeed, in a standard discrete choice model, the representative consumer's surplus is given by $\int_{u_0}^{\infty}(w - u_0)d\Pi_{i=1}^{n} H_i(w + p_i)$, which is identical to the expression above (through integration by parts).

Proof. The expected payoff $EU(u_0)$ can be written as

$$\begin{aligned} EU(u_0) = &\sum_{i=1}^{n} E[V_i + Z_i - p_i|\text{buy from seller } i]Pr\{\text{buy from seller } i\} \\ &+ u_0 Pr\{\text{exercise the outside option}\} - \sum_{i=1}^{n} s_i Pr\{\text{visit seller } i\}. \end{aligned}$$

The first term represents the payoff from trade, the second term captures the payoff from exercising the outside option, and the third term is the expected search cost. By the Envelope theorem, an infinitesimal increase in u_0 affects $EU(u_0)$ only when the consumer exercises the outside option. Therefore,

$$\frac{dEU(u_0)}{du_0} = Pr\{\text{exercises the outside option}\}^{34} = \Pi_{i=1}^{n} H_i(u_0 + p_i).$$

The last equation is true because a consumer exercises the outside option if and only if

[34] See Choi and Smith (2016) for a formal proof.

$u_0 > \max_i W_i - p_i$ by the eventual purchase theorem. Since $CS(u_0) = EU(u_0) - u_0$,

$$\frac{dCS(u_0)}{du} = \Pi_{i=1}^{n} H_i(u_0 + p_i) - 1.$$

When the outside option u_0 explodes to ∞, the consumer never purchases from any seller and thus there is no surplus from search, namely $CS(\infty) = 0$. By the Fundamental Theorem of Calculus,

$$CS(u_0) = CS(\infty) - \int_{u_0}^{\infty} \frac{dCS(u)}{du} du = \int_{u_0}^{\infty} [1 - \Pi_{i=1}^{n} H_i(w + p_i)] dw.$$

■

G Pre-search Information: Proof of Lemma 1

It suffices to show there exists $a' \in (0, 1)$ such that $\partial h(H^{-1}(a))/\partial\alpha < 0$ if and only if $a > a'$. Let Φ denote the standard normal distribution function and ϕ denote its density function. Since $V \sim \mathcal{N}(0, \alpha^2)$ and $Z \sim \mathcal{N}(0, 1 - \alpha^2)$, $F(v) = \Phi(v/\alpha)$ and $G(z) = \Phi(z/\sqrt{1 - \alpha^2})$. Inserting these into equation (2) and differentiating $H(w)$ with respect to α yield

$$H_\alpha(w) \equiv \frac{\partial H(w)}{\partial\alpha} = -\left[1 - \Phi\left(\frac{z^*}{\sqrt{1-\alpha^2}}\right)\right]\left(\frac{w - z^*}{\alpha^2}\right)\phi\left(\frac{w - z^*}{\alpha}\right),$$

where $\partial z^*/\partial\alpha$ can be obtained from equation (1) by applying the implicit function theorem. Differentiating again with respect to w gives

$$h_\alpha(w) \equiv \frac{\partial h(w)}{\partial\alpha} = -\left[1 - \Phi\left(\frac{z^*}{\sqrt{1-\alpha^2}}\right)\right]\left[1 - \left(\frac{w - z^*}{\alpha}\right)^2\right]\frac{1}{\alpha^2}\phi\left(\frac{w - z^*}{\alpha}\right).$$

Now observe that

$$\frac{\partial h(H^{-1}(a))}{\partial\alpha} = h_\alpha(H^{-1}(a)) - H_\alpha(H^{-1}(a))\frac{h'(H^{-1}(a))}{h(H^{-1}(a))}.$$

Let $w = H^{-1}(a)$ and apply $H_\alpha(w)$ and $h_\alpha(w)$ to the equation. Then,

$$\frac{\partial h(H^{-1}(a))}{\partial \alpha} = \frac{-1}{\alpha^2}\left[1-\Phi\left(\frac{z^*}{\sqrt{1-\alpha^2}}\right)\right]\phi\left(\frac{w-z^*}{\alpha}\right)\left[1-\frac{(w-z^*)^2}{\alpha^2}-(w-z^*)\frac{h'(w)}{h(w)}\right].$$

Since $V \sim \mathcal{N}(0,\alpha^2)$ and $Z \sim \mathcal{N}(0, 1-\alpha^2)$, the density of $W = V + \min\{Z, z^*\}$ is

$$\begin{aligned} h(w) &= \frac{1}{\sqrt{1-\alpha^2}}\int_{-\infty}^{\infty}\phi\left(\frac{w-\min\{z,z^*\}}{\alpha}\right)\phi\left(\frac{z}{\sqrt{1-\alpha^2}}\right)dz \\ &= \frac{1}{\sqrt{1-\alpha^2}}\int_{-\infty}^{\infty}\phi\left(\frac{w-z^*}{\alpha}+\max\{r,0\}\right)\phi\left(\frac{z^*-\alpha r}{\sqrt{1-\alpha^2}}\right)dr \end{aligned}$$

where the second line changes variable $r = (z^*-z)/\alpha$. Since $\partial\phi(x)/\partial x = -x\phi(x)$,

$$\frac{h'(w)}{h(w)} = -\frac{w-z^*}{\alpha^2} - \frac{\int_{-\infty}^{\infty}\max\{r,0\}\phi\left(\frac{w-z^*}{\alpha}+\max\{r,0\}\right)\phi\left(\frac{z^*-\alpha r}{\sqrt{1-\alpha^2}}\right)dr}{\alpha\int_{-\infty}^{\infty}\phi\left(\frac{w-z^*}{\alpha}+\max\{r,0\}\right)\phi\left(\frac{z^*-\alpha r}{\sqrt{1-\alpha^2}}\right)dr}.$$

Applying this to the above equation leads to

$$\begin{aligned} \frac{\partial h(H^{-1}(a))}{\partial \alpha} &\propto -1+\left(\frac{w-z^*}{\alpha}\right)^2+(w-z^*)\frac{h'(w)}{h(w)} \\ &= -1+\frac{(z^*-w)}{\alpha}\frac{\int_{-\infty}^{\infty}\mathbb{1}_{\{r\geq 0\}}r\phi\left(\frac{w-z^*}{\alpha}+\max\{r,0\}\right)\phi\left(\frac{z^*-\alpha r}{\sqrt{1-\alpha^2}}\right)dr}{\int_{-\infty}^{\infty}\phi\left(\frac{w-z^*}{\alpha}+\max\{r,0\}\right)\phi\left(\frac{z^*-\alpha r}{\sqrt{1-\alpha^2}}\right)dr}. \end{aligned}$$

The last expression is clearly negative if $w > z^*$. In addition, it converges to ∞ as w tends to $-\infty$. For $w \leq z^*$, it decreases in w because (z^*-w) falls in w and the density $\phi((w-z^*)/\alpha + \max\{r,0\})$ is log-submodular in (w,r). Therefore, there exists w' less than z^* such that the expression is positive if and only if $w < w'$. The desired result follows from the fact that $w = H^{-1}(a)$ is strictly increasing in a. ■

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